OWNERSHIP NETWORKS AND BID RIGGING*

Kentaro Asai, Ben Charoenwong[†]

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Abstract

Using a dataset of public procurement auctions and registered shareholders of all participating firms in Singapore, we study the effects of suppliers' ownership connections on prices and efficiency in the product market. We find identical bidding in the same sealed-bid auction is prevalent and positively correlated with ownership connections among bidders. Moreover, the identical bidding attributable to ownership connections is positively associated with contract price when the lowest bidder wins. Structural estimates based on a model of first-price auctions show the removal of an ownership connection can improve a contractor's cost efficiency by more than it reduces contract price, highlighting how ownership networks hinder competition.

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†Corresponding Author: Ben Charoenwong, National University of Singapore Business School, 15 Kent Ridge Dr #07-69, Singapore 119245, Fax: +65-6779-2083, Phone: +65-6516-5316, Email: ben.charoenwong@nus.edu.sg; Kentaro Asai, Australian National University College of Business and Economics, CBE Building, 26C Kingsley Street, Acton ACT 2601, Australia, Fax: +61-2-612-50744, Phone: +61-2-612-53807, Email: kentaro.asai@anu.edu.au;

1   Introduction

There is a long literature in industrial organization that explores factors facilitating collusion among market participants. Reynolds and Snapp (1986) suggest one such factor is ownership connections between firms in the same industry.[1] They argue when a firm's shareholders also own stakes in its rivals, the firm, representing shareholders' interests, would aim to maximizing the joint surplus of the firm and its *rivals*. By inducing firms to coordinate with one another, ownership connections reduce the competitive intensity of the industry. Since then, the increased intermediation of firm ownership through investment funds has renewed the interest of both academics and policymakers to investigate the effect of common ownership on competition.[2] However, some of the existing research faces criticism due to the methodology and measures used for empirical work. Although the specific setting of common ownership through funds are provocative and important, we need to examine this hypothesis in more settings and industries to better understand the effects of ownership networks on market conduct.

In this paper, we study the effects of ownership connections on prices and cost efficiency in the product market by combining the data of public procurement auctions with information on registered shareholders for all participating firms in Singapore. In the auction setup, whether owners facilitate information flow from one firm to another (Ghosh and Morita, 2017) or influence connected firms to take anti-competitive actions through votes and incentives, both channels would result in some of coordinated bidding strategies. We focus on the submission of identical bids, because it is an unlikely non-cooperative equilibrium if any differences exist in realized costs across firms. In particular, perfectly predicting rivals' bids based only on public information seems technically infeasible in sealed-bid auctions like our samples – instead, we suspect firms more likely share information about their bids.

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[1]See also Bresnahan and Salop (1986), who argue common ownership reduces competition with higher product prices.

[2]The Investment Company Institute reported that the domestic U.S. mutual fund net assets increased from $138 billion in 1986 to $6.8 trillion in 2018.

Consistent with our view, identical bidding in a public procurement auction has been widely viewed as an indicator for the potential presence of coordination (e.g., Mund, 1960; Comanor and Schankerman, 1976). For example, the OECD Competition Committee recommends governments "avoid splitting contracts between suppliers with identical bids and investigate the reasons for the identical bids and, if necessary, consider re-issuing the invitation to tender or award the contract to one supplier only"(OECD, 2009). Moreover, as discussed later in Section 1.2, there is some theoretical foundation behind this strategy as the optimal collusion mechanism in the absence of side payments.

We find (1) a density mass in the distribution of differences exists between two randomly sampled bids at zero, which suggests the prevalence of identical bidding; (2) measures of ownership networks, particularly the presence of common owners and common owners' owners corresponding to second- and fourth-degree connections, are strongly correlated with the number of identical bids across auctions; (3) the effects of ownership connections on the probability of submitting identical bids decrease with the difference in firm capacities, which is consistent with the theory of coordination in repeated games that asymmetries in capacity constraints hinder collusion (e.g., Lambson (1994); Compte et al. (2002));[3] and (4) in auctions where the lowest bid won, the number of identical bids submitted to the auction is positively correlated with the normalized contract amount paid, consistent with the anticompetitive view of identical bidding. In addition, a two-stage least-squares method that decomposes the variation of identical bids into those correlated with ownership networks and those that are orthogonal to ownership networks shows the portion correlated with ownership networks drives the latter result above.

To quantify how much welfare is lost relative to a scenario where ownership networks do not produce collusive incentives, we use a structural approach based on Li et al. (2000) and Krasnokutskaya (2011) to recover the link between the firm's cost and bid. In particular, their econometric frameworks allow us to pool our samples with auction heterogeneity

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[3]Asymmetries in capacity constraints increase the larger firm's incentive to undercut the smaller firm, and limit the smaller firm's retaliatory power (Ivaldi et al., 2003).

that is unobservable for econometricians. Given our data set does not allow us to observe the engineering cost estimates, which are often sources of heterogeneity in public procurement auctions, their frameworks are appropriate for our setting. In addition, Krasnokutskaya (2011) allows us to consider two different types of bidders. Because identical bidding strategy should not be based on the cost drawn from the same distribution as the cost of a competitive bidder, her framework that accommodates asymmetric bidders particularly suits to our setting. One caveat is that we focus only on auctions in which lowest bidders win while we assume bidders know lowest bidders win for these auctions. This focus and assumption is required for our procedure, because their frameworks deal with first-price sealed-bid auctions.

Utilizing their frameworks, we estimate the cost distributions of firms and simulate the auction outcomes of a counterfactual world where firms bid in the absence of the effects of ownership connections. When solving the bidding functions of asymmetric bidders as the system of differential equations, we use both shooting and projection algorithms of Bajari (2001) to avoid the instability issue associated with the shooting algorithm. By comparing the simulation results of control and counterfactual groups, we find the removal of one ownership connection reduces a winning bid by 0.4 to 7.6% and a winner's cost by 4.1 to 7.7% of the winning bid.

Our paper contributes to the existing literature by empirically examining the effects of ownership connections through the lens of public procurement auctions. Our use of public procurement auction data is advantageous for testing the effects for several reasons. First, ample intra-industry variation of ownership concentration exists due to a large number of auctions tendered by the same procurer within a given time period. Specifically, a different set of firms participate in different tenders, providing rich variation in ownership concentration across auctions. Second, our primary measure of anti-competitiveness is the incidence of bid rigging, not price itself. Although the incidence of bid rigging is hardly explained except for the presence of tacit or explicit collusive incentives, prices could be increased by various factors such as increases in firms' production costs even in competitive markets.

Then, estimating the effects of ownership connections through price-concentration regressions is relatively susceptible to interpretation problems. On the contrary, our use of bid rigging allows us to reduce the possibility of such problems. Third, our data set captures various industries ranging from goods to services due to the comprehensive nature of public procurement. Whereas the recent empirical evidence is often generated from the dataset of specific industries such as airlines and banks, we provide empirical implications that can be applied to a wider range of industries. Last, we can acquire the welfare implications of ownership networks using the rich literature on structural econometric methods in auctions.

1.1 Related Literature

Our paper is most related to the empirical research of common-ownership effect that straddles both finance and industrial organization literatures. For example, Azar et al. (2018) provide the pioneering evidence that common institutional ownership predicts higher route-level airline prices. This result suggests an investor's incentive to diversify can end up reducing product market competition through reduced governance incentives. However, Kennedy et al. (2017) criticize their findings by showing the absence of economic micro-foundations behind their empirical specifications in a differentiated-product oligopoly. As in Azar et al. (2018), Azar et al. (2016) show evidence that ownership concentrations among banks are correlated with interest rates and service fees.[4] Cici et al. (2015) study the effects of common ownership on syndicated loan market interactions and find borrowers and lenders that are commonly held by an institutional block-holder tend to do more business together going forward. We add to the literature by providing alternative evidence of common ownership effect using public procurement auction data. In particular, our findings suggest even within the same "market," defined as participation in the same public procurement auctions, common ownership can reduce competition. This implication complements Mathews (2006), who rather suggests common ownership results in firms avoiding the same market.

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[4]See also surveys by Backus et al. (2019) and Schwalbe (2018).

Our paper also provides an underestimation of the effect of a broader firm network mediated by common stakeholders. An existing literature has documented the role of informal interactions and networks in transmitting both information and anti-competitive practices, both of which are not observed in our sample. For example, Agarwal et al. (2019) show corporate directors of real estate companies visit golf courses more regularly after land-scale announcements. They also find the more frequent visits correlate with lower winning bids and lower revenues to the government.[5]

Moreover, our findings are related to the prior literature of collusion detection. Porter and Zona (1993) study bidding patterns to detect collusion in public procurement auctions. Athey et al. (2008) study bidder competitiveness through a structural approach. Bajari and Ye (2003), Kawai and Nakabayashi (2014), and Chassang et al. (2019) propose the method of distinguishing collusion from competition in auction setup through a data-driven approach without prior knowledge of potential colluders. Baldwin et al. (1997) examine whether price variations, after one controls for demand conditions, are better explained by collusion or, alternatively, variations in timber-supply conditions. Harrington (2005) reviews methods for detecting cartels and distinguishing collusion from competition. Whereas these papers propose the method of detecting collusion through bidding patterns, our paper provides a clue to the identity of potential colluders by focusing on firm networks. In particular, our findings imply regulators interested in monitoring competition should not only exploit the information of common ownership, but also examine indirect connections through common owners' owners. Even with the threat of government enforcement, LaCasse (1995) shows the

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[5]In addition, the finance literature has also documented the role of informal connections on corporate governance. Cohen et al. (2008) find portfolio managers invest more in firms in which the manager went to the same graduate business school, and also tend to perform better in those investments. Kuhnen (2009) show that business networks between mutual fund directors and advisers mitigate agency conflicts. Cesare and Tate (2010) find that firms with more powerful CEOs are more likely to appoint directors with ties themselves, reducing firm value. Nguyen (2012) shows that board members connected with the CEO are less likely to force a CEO turnover following poor performance. Engelberg et al. (2012) find that firms that borrow from socially connected banks have lower interest rates, but does not appear related to sweetheart deals, especially because subsequent firm performance improves following a connected deal. Ishii and Xuan (2014) find that mergers with prior target-acquirer social connections between directors and senior executives lead to more negative abnormal returns to the acquirer upon the merger announcement.

government may not be able to distinguish bid rigging through the evaluation of bids alone. Along this line, our paper provides evidence that additional data may be useful in aiding detection of anti-competitive behavior.

In addition, our paper is related to the empirical literature on colluding firms' strategies. As discussed by Levenstein and Suslow (2006) and Levenstein and Suslow (2011), successful collusion divides cartel profits while suppressing incentives for cheating. Compared to identical bidding, bid rotations provide a clear market-sharing rule, but the coordination costs will be higher because the cartel must agree on a division of cartel profits. Depending on the cost and benefit of agreeing on a clear division of cartel surplus, the range of bidding strategies from identical bidding to bid rotations is expected to occur.[6] Recent papers such as Chassang and Ortner (2018), Kawai and Nakabayashi (2014), and Chassang et al. (2019) study Japanese public procurement auctions. Pesendorfer (2000) studies school milk contracts in Florida and Texas during the 1980s. Although these papers document isolated bidding, refraining from bidding through side payments, and market division, our paper rather reveals the prevalence of identical bidding in Singaporean public procurement auctions.

Lastly, our findings suggest identifying ownership networks helps raise the probability of detecting corporate wrongdoing. Although common owners benefit from firms' collusive behavior, the majority of shareholders or the society may want firms to compete and make efforts for raising market shares. Then, firms' collusive behavior do not reflect the interests of these shareholders or the society. The prior literature focuses on firms' internal structure such as CEOs' connections with top executives and directors (e.g., Khanna et al. (2015) and Chidambaran et al. (2012)), easiness to divert income (e.g., Desai et al. (2007)), board structure (e.g., Beasley (1996) and Agrawal and Chadha (2005)), corporate lobbying (e.g., Yu and Yu (2011)), and executive compensation (e.g., Burns and Kedia (2006) and Efendi et al. (2007)) as the driving factors for corporate scandals. Our paper suggests inter-firm ownership connections can also motivate firms to deviate from maximizing shareholders'

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[6]Apart from auctions, to explain recent collusive practices, Harrington and Skrzypacz (2011) also characterize a stable collusive agreement when firms' prices and quantities are private information.

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value or social welfare.

1.2   Bid Rigging as the Optimal Collusion Mechanism

Before describing our empirical framework, we briefly discuss the theoretical background of identical bidding as the optimal strategy for collusive bidders. McAfee and McMillan (1992) contribute to our theoretical understanding of identical bidding. Their analysis shows the best collusion mechanism for a first-price sealed-bid auction is one in which all bidders submit identical bids if side payments are prohibited.[7] If side payments were allowed, cartel members would be willing to allocate a project to the most efficient bidder to maximize cartel surplus. Then, cartel members would have an incentive to truthfully report their costs. However, if side payments are prohibited, only the winning bidder can receive cartel surplus, and hence cartel members have an incentive to misreport their costs to become the winner of the auction. Because the revelation principle suggests the optimal mechanism is incentive-compatible without loss of generality, the mechanism requires the project be awarded randomly to induce truthful reports.[8] Because the procurer assigns the project to suppliers that bid the same price with equal probability, the submission of identical bids works as a randomization device and induces truthful reports.

One important implication from this analysis is that projects are randomly allocated to one of identical bidders, meaning that the most efficient member may not undertake the project. On the other hand, if side payments are available, the most efficient member bids against non-cartel members based on her cost on behalf of the cartel — even if the other members bid, their bids are set to be above her bid. In this way, whenever the cartel wins the auction, the most efficient bidder undertakes the project.[9] Thus, even if both mechanisms raise winning bids, their implications on contractors' cost efficiency are quite different. If

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[7]Athey et al. (2004) also attain a similar result for a repeated procurement auction.

[8]See Myerson (1981). The incentive-compatible mechanism requires that any deviation from the prescribed bidding behavior be punished ex post. We note such deviation can be detected ex post in our sample auctions. As discussed earlier, all bidders and their corresponding bids in a procurement contract of Singapore are made public for six months, as well as the identity of the winning bid.

[9]See Marshall and Marx (2007), regarding the optimal collusion mechanism when the cartel can control bids.

side payments are prohibited and identical bidding is rationalized as the optimal collusion mechanism, we anticipate larger deterioration of cost efficiency. This conjecture motivates us to assess the effects of ownership connections on the winning bidder's cost using structural framework.

Unfortunately, McAfee and McMillan (1992) focus on all-inclusive cartels — the best mechanism for partial cartels in the absence of side payments seems an open question.[10] Incentive compatibility seems to require cartel bids be identical to give an equal chance of winning the auction even in the presence of non-cartel bidders, but the optimal mechanism for partial cartels in the absence of side payments has not been fully characterized.

We therefore empirically examine whether identical bidding has anti-competitive effects. As discussed later, we find the number of identical bids is positively associated with the winning bid when the winner of the auction is the lowest bidder. Moreover, we document the effects of ownership connections on the probability of submitting identical bids are larger for a pair that is similar in capacity, which is consistent with the theoretical prediction that symmetric firms are more likely to coordinate in repeated games.

2   Public Procurement Auctions in Singapore

Since 2015, every government agency in Singapore except for the Expenditure and Procurement Policies Unit, Defense Science and Technology Agency, and Infocomm Media Development Authority announce their purchases through a one-stop online portal hosted by the Ministry of Finance called GeBIZ, standing for Government Electronic Business. For the most part, different government agencies operate independently on the GeBIZ portal.[11]

Procurements are classified as one of the following: small-value purchases (up to S\$5,000 or around US\$3,700), for which reporting requirements only include the invoice and total amounts purchased; invitations to quote (S\$3,001 to S\$70,000 or around US\$2,200 to

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[10]We note Marshall and Marx (2007) study partial cartels in the presence of side payments.

[11]However, different departments within a government agency that run separate simultaneous auctions may affect how the agency as a whole awards contracts (see section 2.2).

US\$52,000), for which procurement includes online solicitation of qualified bids through GeBIZ; or invitations to tender (above S\$70,000 or around US\$52,000), for which a similar procurement requirement applies. Information on all types of procurements are available on GeBIZ to the public, and the only differences are the levels of qualifications or types of bidders that may participate in the auction.

Any publicly listed or private company may register to be a government supplier, subject to a registration fee ranging from S\$300 to S\$5,000 and due-diligence requirements based on the line of business. For example, businesses related to information technology are subject to more due diligence with respect to cybersecurity than companies wishing only to sell photography services. Participation in larger contracts also requires more certifications and more stringent due diligence.

The auctions have a solicitation period, typically around 30 days. Eligible companies blindly submit bids and may update their bids through time without seeing competitors' bids. When the auction closes and the solicitation period ends, the GeBIZ platform publicly discloses all final bids submitted by each bidder. The winning bidder provides the good or service for the government at the price it submitted. Although almost all government procurements are awarded, a handful are closed with no award if no qualified bidders are participating in the auction. This scenario may occur if no bidders are present, if the government agency determines the number of bidders is insufficient, or if the agency perceives that none of the bidders in the auction have the capacity to service the contract fully.

[Table I Around Here]

Table I shows the breakdown of total auction revenues by the top 10 government agencies by number of auctions and total expenditure, and also shows the breakdown of auctions into goods, services, and construction services. The Housing and Development Board, the government entity responsible for all public housing in Singapore that provides housing for over 85% of Singapore citizens, accounts for almost half of government expenditures in our

sample.[12]

2.1 Competitive Law

All public procurements are subject to the Government Procurement Act (GPA). In adherence to the Agreement on Government Procurement from the World Trade Organization, the GPA outlines the requirements for public procurements in Singapore that aim to foster fair, competitive, transparent, and non-discriminatory conditions for government purchases of goods, services, and construction works. In addition to the GPA, the Competition Act provides further guidelines and regulations for procurement. These regulations are enforced by the Competition & Consumer Commission of Singapore (CCCS).

Although the GPA does not explicitly place any minimum number of bidders on an auction, Section 34 of the Competition Act prohibits agreements, decisions, and practices that are anti-competitive. The commission provides a website outlining the basic anti-competitive practices, including price-fixing, bid rigging, market sharing, and production control. Although they do not explicitly outline penalties or enforcement practices, they "encourage all businesses to set their prices independently."[13] Anecdotally, anti-competitive practices that have been publicized in the media have resulted in fines, termination of contracts, and jail-time for related individuals. In addition, the Ministry of Communications and Information provides additional information regarding the public procurement system on its website. The guidelines state,

> Government agencies must assess the reasonableness of the bids regardless if
> a single or several bids have been received. When recommending the award of
> a tender/quotation based on a single bid, officers are required to justify to the
> Approving Authority why the single bid is considered competitive or reflective

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[12]In addition, up until mid 2017, the National University of Singapore still participated in the GeBIZ program prior to rolling out their own separate procurement service, with auctions from March 2017 through July 2017 accounting for S$864 million. Most of the auctions involve administration and training services, followed by IT equipment and laboratory equipment.

[13]From the CCCS website.

of fair market value. For example, they may have performed independent checks or consulted experienced buyers. If no reasonable bid has been received, agencies may call for a fresh quotation/tender, after revising their requirements, if necessary.

2.2   Award Criteria

Due to the wide variety of contracts that are awarded through GeBIZ auctions, note the procurement contracts are not awarded purely based on price. The Singapore government claims they use a criterion called "value for money," which evaluates all bids along multiple dimensions, not just price. These dimensions include the bidder's ability to meet the contract requirements (e.g., the government must believe a company is able to fulfill the quantity and quality levels expected from the contract requirements), quality and reliability of products and services, non-upfront maintenance costs, operating costs, warranty clauses, and demand aggregation among different departments within a government agency. The official government website provides a list of myths pertaining to procurement auctions, with the following:[14]

> Myth 2: The lowest bidder always wins.
>
> This is not true. While suppliers' bids are evaluated on their value for money, it does not mean contracts are always awarded to the lowest quote. While price is a key consideration in evaluation, Government agencies check if bids have complied with all the requirements in the tender specifications, as well as evaluate other factors such as quality of the goods and services, timeliness in delivery, reliability, and after-sales support. Roughly half of all procurements are not awarded to the lowest quote.

Despite price not being the only criterion for evaluation, it still matters. In addition, we expect that auctions for commodity goods and services are particularly more subject to price

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[14]https://www.gov.sg/factually/content/quality-vs-value-how-does-government-evaluate-tenders

competition than those with a wide range of quality. In our sample, around 50% of auction winners were those with the lowest bid, consistent with what the government reports.

In Appendix B, we use both linear models as well as a random-forest model to back out what variables most likely drive whether the lowest bidder wins. The classification accuracy ranges between 67.1% and 69.4%. The random-forest model suggests the most important auction characteristic is the number of bidders in an auction, with more bidders leading to a lower probability that the lowest bidder wins, followed by the auction category and the number of outstanding open auctions that the government agency has either within a 30-day or 10-day window around the auction date. The full list of variable importances is shown in Figure A.IV, and the relative performances of different classification models are shown in Table A.III for the linear models, with the receiver operating characteristic (ROC) plot shown in Figure A.V.

3   Data & Methodology

3.1   Data

Our unique dataset permits the empirical analyses of firm ownership and auction results. In Singapore, all bidders and their final bids in a procurement auction are made public for six months, and the winner is identified. After six months, information on non-winners is removed, and only the data on winning bids and awarded amounts remain public. Because of this policy, we build our policy in real time, starting in 2017 and going back six months, and then updating the data every six months. Our final sample covers September 21, 2016, to April 2, 2018, starting a little over one year from the GeBIZ system's initial rollout in 2015. Our sample contains more than 24,000 auctions, totaling over S\$16.5 billion (around US\$12 billion) in government expenditures. Annually, the Singapore government reports that transactions on GeBIZ account for around 50% of the total annual Singapore government

expenditure.[15]

Of the total public procurement auctions since September 2016, the average contract amount is a little over S$678,000, with the median contract amount being only a little over S$17,800. A large right skew occurs in the data, because auctions may range from purchasing of single computers all the way to building a new subway system or public housing units.

In addition to the auction data, we acquire the shareholder registry and financial-statement data for both publicly listed and private companies from DC Frontiers Pte Ltd (Handshakes), one of four authorized information resellers licensed by the Accounting and Corporate Reporting Agency (ACRA).[16] The data include registered shareholder and officer information, including unique identifiers, for every company and financial-statement data for large companies with more than S$1 million in revenues.[17] A private limited company may have between 1 and 50 shareholders while public companies have no restrictions on the number of shareholders it has.[18] For both cases, all registered shareholders submit a passport copy or identity card, proof of residential address, and a brief professional background for registry with the ACRA. Where the shareholder is another company, ACRA requires its certificate of

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[15]The total Singapore government expenditure includes spending through the military and other military-related research and development that does not occur on GeBIZ.

[16]Handshakes aggregates, cleans, and maintains data for sale to third parties and end users. See https://www.acra.gov.sg/components/wireframes/howToGuidesChapters.aspx?pageid=1262#1277 as of May 11, 2018. ACRA is the government entity handling corporate data, similar to the Securities and Exchange Commission (SEC) in the United States. However, unlike the SEC, ACRA does not enforce securities law. That function is allocated under the Monetary Authority of Singapore.

[17]Shareholders must be registered to receive dividend payouts. If end investors own a firm's stock through a brokerage, the brokerage becomes the registered shareholder (as a company) and distributes the dividends accordingly.

[18]Most bidders in our sample are private companies whose stock does not trade often. Thus, we believe the shareholder information is fairly up to date. Nonetheless, apart from annual general meetings violations, ACRA's website list the next two most common compliance offenses for companies to shareholder information accuracy, Sections 82(1) and 82(2) of the Company Act. Section 82(1) states the offense as, "a person who is a substantial shareholder in a company fails to give notice in writing to the company stating his name and address and full particulars (including unless the interest or interests cannot be related to a particular share or shares the name of the person who is registered as the holder) of the voting shares in the company in which he has an interest or interests and full particulars of each such interest and of the circumstances by reason of which he has that interest." Section 82(2) mandates a company notice filing within two days of a person becoming a substantial shareholder after October 1, 1971. Prior to that, the grace period was one month. Stale information, that cause us to observe a connection when there is none would introduce a downward bias, making our estimates and thus counterfactual simulation results more conservative.

registration, state of incorporation, official company profile, ownership structure that identifies the ultimate beneficial owners and shareholders of the company, and an authorization resolution passed by directors for shareholder registration.

In addition to shareholders, the data include information on directors, and other company officials through time, permitting us to study the impact of different types of connections as well. For public corporations, we do not see the name of every individual shareholder, but only observe shareholders with at least 1% ownership and those that paid up capital since the inception of the company if they are current shareholders. From this observation, we find connections between pairs of firms up to the fourth degree, shown below.

3.2   Defining Ownership Networks

To identify common-ownership networks, we first consider the symmetric adjacency matrix A, an $n \times n$ matrix where each element $A(i,j)$ represents the number of linkages from entity i to entity j, and n is the number of unique entities, comprising both individuals and businesses.[19] We treat all connections as static and do not consider new or expiring relationships during our sample period, because our sample period spans less than two years.

For an adjacency matrix S, we calculate k^{th}-degree connections between two firms as the $(m,n)^{th}$ element of the adjacency matrix raised to the power k subtracted by the $(m,n)^{th}$ element of the adjacency matrix raised to the powe $k-1$, $S^k(m,n) - S^{k-1}(m,n)$. Raising an adjacency matrix to a power k captures the number of walks up to k-steps through a particular network relationship from node m to node n. Subtracting the matrix S^{k-1} isolates the number of walks that are k-steps and not less. In the reduced-form empirical specifications, separating out k-step specific walks rather than using the cumulative walks up to k-steps also reduces collinearity in the explanatory variables.

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[19]In Singapore, all businesses have a unique entity number (UEN) that can be publicly searched, including all individuals participating in auctions as service providers who are required to register at least as sole proprietorships and are assigned a UEN. Apart from this number, we have the full legal names of registered shareholders. Although we do not have unique identifiers like their citizenship or passport number, we confirmed that we do not have any duplicates in our sample.

The nature of these connections implies every odd k-step walk has to go through a company. Figure I visualizes these connections up to the fourth degree. For two arbitrary firms A and B participating in the same auction, a first-degree connection is if A is a shareholder of B, a second-degree connection is if A and B have the same common shareholders (that may be either people or firms), a third-degree connection is if A is owned by another firm C whose shareholder is also a shareholder of B, and a fourth-degree connection is if A is owned by firm C while B is owned by another firm D and firm C and D have the same shareholders (that may be either people or firms).

[Figure I Around Here]

3.3 Reduced-Form Methodology

We begin our analyses by documenting the association between firm ownership connections and the prevalence of identical bids. We use this association later on in our structural analysis.

3.3.1 Auction-Level Analysis

We use a two-stage least-squares procedure as a method for assessing the anticompetitive effects of ownership connections. It extracts the impact of the number of ineffective bids on the winning bid that is driven by variations in ownership connections among bidders. In the first stage, we estimate the effects of ownership connections among firms participating in the same auction on the competitive intensity of bidding in that auction, controlling for auction-specific characteristics and the fact that bids tend to be round numbers. In the second stage, we study the impact of bidding competitiveness predicted by the first step regression on the expenses to the government. We discuss the exact specification and construction of different variables of interest below.

For each auction j with a set of bidders $B(j)$, arbitrarily indexed from $i = 1,...,N_j$, we count the number of ineffective bids N_j^I, defined as the number of duplicative bids in the

auction. In our setting, each bidder $b \in B(j)$ can only submit one bid, so $N_j^I = N_j - N_j^E$, where N_j^E stands for the number of effective bids and is the number of distinct bids in auction j. Under one interpretation, the number of ineffective bids N_j^I reflects the extent of identical bidding in an auction. Under another interpretation, this variable measures the number of firms that do not affect the competitive intensity of bidding.

For ownership connections, we count the number of shared connections among auction participants based on current shareholders. For each arbitrary pair of firms (m, n), we define the presence of a connection of degree k as the indicator $1(S^k(m,n) - S^{k-1}(m,n) > 0)$, where S^k is the symmetric current-shareholder adjacency matrix raised to the power k and $S^k(m,n)$ is the mth row and n^{th} column of matrix S^k. We aggregate the indicators from every possible pair of bidders in an auction to form an auction-level connectedness by defining the number of connections as

$$NC_j^k = \sum_{(m,n) \in B(j)} 1(S^k(m,n) - S^{k-1}(m,n) > 0),$$

where $B(j)$ denotes the set of bidders that participate in auction j. As discussed before, we consider up to degree $k = 4$.

We next measure the tendency of bid rounding at auction level. Specifically, the roundedness measure R_j is defined as

$$R_j = \frac{1}{N_j} \sum_{i=1}^{N_j} 1(b_i \equiv 0 \mod 10^{k_j}),$$

where N_j is the number of bidders in auction j, i indexes each bidder with bid b_i, and k_j is the statistical mode of the magnitude of all the bids in an auction j minus one. This specification of the roundedness assumes the rounding of bids tends to occur immediately below the order of magnitude of the bid. For example, we assume an unrounded bid of S\$10,201 tends to be rounded to S\$10,000, and S\$4,282 tends to be rounded to S\$4,300. In accordance with this premise, if the two bids submitted to auction j are S\$7,500 and S\$6,840, for example, $k_j = 2$ and $R_j = 0.5$ because S\$7,500 is divisible by 10^2 whereas S\$6,840 is not.

In the first stage of the analysis, we consider a regression specification of the form:

$$N_j^I = \sum_{k \in K} \beta^k NC_j^k + f(Z_j) + g(R_j) + \epsilon_j, \tag{1}$$

where K is the set of degrees considered in the regression, N_j^I is the number of ineffective bids in auction j, NC_j^k is the total number of pair-wise connections among all bidders of degree k in auction j, β^k is the coefficient of interest and captures the effect of one pair-wise k^{th} degree connection of any pair on the number of ineffective bids, $f(Z_j)$ is a semi-parametric function of Z_j, specified as number-of-bidder and procurer-auction category fixed effects, and $g(R_j)$ is a cubic polynomial of bid rounding R_j.[20] The inclusion of $g(R_j)$ controls for the effect of simple bid rounding on the number of ineffective bids. We cluster standard errors by the procuring government agency, allowing for within-agency correlations such as awarding rules and needs.[21]

In the second stage of the analysis, to assess the impact of identical bidding on the award amount, we investigate the link between the number of ineffective bids and the amount of the winning bid that the government pays. In our regression, we use the winning bid that is normalized by by the average bid of the auction, $Amount_j$. $Amount_j$ reflects the planned expenditure of the government and revenue for the bidding firm. Considering identical bidding is likely to be the optimal collusion mechanism for a first-price sealed-bid auction, we limit our attention to auctions in which lowest bidders win.[22] We regress $Amount_j$ on N_j^I using the specification

$$Amount_j = \gamma \widehat{N_j^I} + f(Z_j) + g(R_j) + \epsilon_j, \tag{2}$$

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[20]Figure A.III in Appendix A shows the histogram of trailing units. We find that although a handful of bids do not end in an integer, a mass is present at 0, suggesting firms round their bids at least to the nearest 10's unit.

[21]Whether an agency awards a firm may depend on whether the firm can simultaneously fulfill multiple procurement contracts.

[22]For the theoretical background of identical bidding as the optimal collusion mechanism, see section 1.1.

where the control variables mirror those in the first stage, and $\widehat{N_j^I}$ is instrumented by the number of second- and fourth-degree ownership connections. In the above equation, we re-define $f(Z_j)$ as quadratic controls for the number of bidders and procurer-auction category fixed effects, though we keep specifying $g(R_j)$ as cubic controls of bid roundedness. The coefficient γ captures the effect of one ineffective bid submitted to the auction on the winning bid, which is identified through variations in the number of ineffective bids induced by ownership connections and variations in the winning bid. As with the first-stage analysis, we cluster standard errors by the procuring government agency, allowing for within-agency correlations such as awarding rules and needs.[23]

We also run the corresponding regression of *Amount$_j$* on the uninstrumented number of ineffective bids (N_j^I). By comparing the coefficient γ across the two regressions, we assess the contribution of ownership connections in the observed correlation between the number of ineffective bids and the winning bid.

3.3.2 Pair-Level Analysis

We also investigate the possibility of identical bidding as a collusive strategy of connected firms in dynamic context. Lambson (1994), Compte et al. (2002), and Ivaldi et al. (2003) argue asymmetries in capacity constraints hinder collusion in repeated games, because smaller firms are less able to retaliate against a larger firm; thus, the larger firm may undercut the smaller firm without large consequences. Then, this finding suggests if identical bidding is used by connected firms as a collusive strategy, it is less likely to occur between connected firms with large capacity difference. To test consistency with this view, we conduct a pair-level analysis of identical bidding behavior for firms that have participated in the same auction at least once.

In particular, we investigate how asymmetries in size moderates the effects of ownership connections on the probability of submitting identical bids. For this analysis, we focus on

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[23]Whether an agency awards a firm may depend on whether the firm can simultaneously fulfill multiple procurement contracts.

second-degree connections because of data availability.[24] We then regress the probability of submitting identical bids for pair p, I_p, on the dummy variable that indicates the presence of second-degree ownership connection, C_p^2, and its interaction with the proxy for asymmetries in capacity constraints, X_p:

$$I_p = \alpha + \eta C_p^2 \times X_p + \beta C_p^2 + \delta X_p + \epsilon_p, \qquad (3)$$

where η is the coefficient of interest capturing the differential effect of second-degree ownership connection by asymmetries in capacity constraints, and β captures the effect of second-degree ownership connection in the absence of asymmetries in capacity constraints X_p. To measure asymmetries in capacity constraints, we consider using both the difference in revenue and the difference in asset size between each pair of firms. We also report the regression results for the limited set of pairs that have participated in the same auction at least five and 10 times, because the pairs that have often participated in the same auction are more likely to play in repeated games.

3.4   Summary Statistics

In our auction-level data, we observe 24,628 auctions in total with bids from 82,604 unique bidders. The average number of bidders is around six per auction, with a range from one to over 30. Most of our analyses only use auctions with more than one bidder, reducing the number of observations to 22,098 auctions. Of these auctions, the average number of ineffective bids in an auction is 0.45, with 9.8% of auctions having at least one ineffective bid. The average number of first-degree, second-degree, third-degree, and fourth-degree ownership connections per auction is 6.3 basis points, 8.7 percentage points, 0.45 basis points, and 48 basis points, respectively.

In our pair-level data, we observe 225,295 pairs that participate in the same auction at

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[24]We observe pair-wise firm characteristics for only a handful of pairs that share ownership connections at other degrees (1, 3, and 4).

least once in total. On average, a pair of firms participate in the same auction 2.8 times and submit identical bids for 1.7% of the auctions in which they jointly participate. Of all the pairs, the probability a first-degree connection is 0.40 basis points (9 pairs), a second-degree connection is 7.59 basis points (171 pairs), a third-degree connection is 0.04 basis points (1 pair), and a fourth-degree connection is 0.72 basis points (16 pairs). Among all these pairs, we observe inter-firm characteristics for 6,592 pairs, which we later use to test whether capacity differences affect the relation between ineffective bids and connections.

4   Reduced-Form Results

4.1   Bid Differences in Auctions

To see the likelihood of identical bidding occurring in public procurement auctions in Singapore, we construct the distribution of the gap between two arbitrary bids from the same auction. For each auction in our sample, we sort bidder names in increasing order and select the first and second bids. Then, we compute the gap between the selected bids that is normalized by the second bid.

[Figure II Around Here]

The histogram of randomly sampled differences between two bids in each auction shown in Figure II shows a probability mass at zero, suggesting bid rigging may exist and be pervasive in the public procurement auctions. Moreover, it reveals the density at zero is *discontinuously* larger than the density immediately before and after zero, though the bid difference is smoothly distributed except at zero. If suppliers independently drew costs from smooth distributions, their cost difference would be smoothly distributed. And if cost differences were smoothly distributed, their bid differences would also be smoothly distributed, given that the standard competitive equilibrium bidding strategy suggests their bids are strictly monotonic and continuous in their costs. In this respect, our finding seems

inconsistent with a competitive bidding outcome. The bottom panels of Figure II also presents the corresponding distribution for the subset of auctions for goods and services, respectively. We observe the consistent pattern across different types of items.[25] This finding suggests identical bidding non-trivially occurs in our sample.

4.2 Ineffective Bids and Connections

Table III reports the auction-level regression result of equation (1). Columns (1) and (3) show the estimates of β^1 and β^3 from Equation 1 are relatively small in magnitude and statistically insignificant even at 10% level. We consider these coefficients are not precisely estimated, given the limited variations in the number of first- and third-degree ownership connections across auctions. Column (2) of the table shows the estimate of β^2 is 0.557, which is statistically significant at 1% level. This estimate suggests having another second-degree connection increases the number of ineffective bids by 0.557. Similarly, column (4) of the table shows the estimate of β^4 is 0.379, which is statistically significant at 1% level. This estimate implies having firms with one fourth-degree connection increases the number of ineffective bids by 0.379. Columns (5) shows the relation between ownership connections and identical bidding where we control for all degrees of NC_j^k. Overall, the estimates for β^2 and β^4 remain similar to the above estimates. Columns (6) through (9) show the relation between ownership connections and identical bidding while conditioning on auctions with more participants. We find the estimates for β^2 and β^4 remain positive and statistically significant at 1% level for these specifications.

Table A.I in Appendix A presents the corresponding results by the type of items to be procured, and also shows the sensitivity of our main point estimates to different fixed effects. Overall, we find the positive effects of second- and fourth-degree ownership connections on the number of ineffective bids are found in both goods and services auctions, regardless of specifications. This result suggests the prevalent importance of ownership connections across

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[25]Figure A.II in Appendix A shows the histograms based on the number of bidders in the auction and documents a similar probability mass at zero across all filters considered.

different types of industries.

[Table III Around Here]

Table IV reports the pair-level regression result of equation (3). Columns (1) through (3) show the estimates of η when capacity difference is measured by revenue gap, whereas columns (4) through (6) present the corresponding estimates when capacity difference is measured by gap in asset size. For each measure of capacity difference, we restrict our sample pairs by the number of joint participations. Overall, we find the estimates of η are negative for all specifications and statistically significant at 5% level for most of the specifications. These estimates suggest the higher probability of identical bidding by connected firms of a similar size. This result is consistent with an intention to coordinate in repeated games.

[Table IV Around Here]

4.3 Alternative Possibility

One alternative interpretation of identical bidding is that connected firms tend to have distinct but similar cost levels. If firms simply round their bids, connected firms are likely to submit identical bids even if their costs are distinct. Then, some of the estimated effects of ownership connections on the number of ineffective bids can capture the channel mediated through cost similarity. Indeed, Appendix A.III shows the prevalence of rounded bids.

To rule out this channel, we re-estimate equation (1) while controlling for the interaction term of the auction's bid rounding level R_j and the number of ownership connections NC_j^k. Then, the coefficient on NC_j^k is interpreted as the effects of ownership connections on the number of ineffective bids in the absence of bid rounding. As reported in Table A.II of Appendix A, the estimated coefficient on NC_j^k is very close to the corresponding estimates in columns (2) and (4) of Table III. From this finding, we conclude our benchmark estimates for the effects of ownership connections on the number of ineffective bids are not driven by the channel mediated through cost similarity.

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4.4 Ineffective Bids and Bid Amounts

Table V reports the auction-level regression results of equation (2) in the endogenous regression specification and instrumental variables (IV) specification. Columns (1) and (2) present the results for all eligible auctions, columns (3) and (4) only use auctions for goods, and columns (5) and (6) only use auctions for services. Columns (1), (3), and (5) present results for the uninstrumented number of ineffective bids, whereas columns (2), (4), and (6) present corresponding results for the number of ineffective bids instrumented by second- and fourth-degree ownership connections. Column (2) shows the estimate of γ is 0.045, which is statistically significant at 10% level. The first-stage F-Statistic (26.335) exceeds the Stock-Yogo threshold of 10 for an instrumental variable estimation bias of up to 10%. An increase of one identical bid related to shared owners (second-degree connections) or owners' owners (fourth-degree connections) raises the award amount by 0.045 of the average bid in the auction. Column (4) shows the corresponding estimate for goods is 0.176 of the average bid, which is statistically significant at 5% level. As before, the first-stage F-Statistic (31.924) exceeds the Stock-Yogo threshold and do not appear to be weak instruments. On the other hand, column (6) shows the corresponding estimate for services is almost zero.

The results of the corresponding regressions with the uninstrumented number of ineffective bids are reported in columns (1), (3), and (5), respectively. Unlike the previous regressions, the estimates of γ range between 0.04 and 0.05, which is relatively uniform across the types of items to be procured and statistically significant at 1% level for all types. Because the estimates of γ in the IV estimate (column (2)) and the corresponding estimate for the uninstrumented variable (column (1)) are similar, the observed correlation between the number of ineffective bids and the winning bid is mostly driven by variations in the number of ownership connections. Our estimates also suggest ownership connections play a more important role in goods auctions than in services auctions in terms of their ultimate influences on award amounts, as the estimate of γ in the IV estimate for auctions (column (4)) exceeds that for services (column (6)). We, however, note the estimate of γ in the

IV estimate for services (column (6)) is unlikely to be precisely estimated, given that the first-stage F-Statistic (8.995) is below the rule-of-thumb threshold of 10. We cannot rule out the possibility that the imprecise estimate of the first-stage regression may drive the weak estimate of γ at the second stage.

[Table V Around Here]

5 Ownership Networks and Auction Efficiency

The reduced-form analysis provides suggestive evidence on the role of ownership networks in collusive bidding, but provides little implication on an auction's efficiency. We therefore use a structural framework to recover the missing link between a firm's cost and bid and identify the distribution of a firm's cost.[26] By simulating the auction outcomes of a counterfactual world with no relationship between ownership connections and bidding behavior, we are able to assess the effect of being in ownership networks on the cost of the winning contractor, which is interpreted as the measure of an auction efficiency.

5.1 Structural Analysis

For the appropriate choice of structural framework, we consider two issues. First, our data set consists of auctions with various project sizes.[27] Then, to estimate the cost component that is relevant to bidding strategy, we need to extract out the auction-specific component from observed bid distribution. Second, collusive and competitive bidding strategies are likely to be asymmetric. To see how, we regress effective bids on the indicator for identically

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[26]Bajari and Hortaçsu (2005) provide evidence, based on first-price auction experiments, that structural estimation provides reasonable estimates of bidder valuations, which can then be used for counterfactual analysis. Given this finding, our approach uses structural estimation for the counterfactual analysis in order to have a wider implications of reduced-form results.

[27]Whereas the procurer's cost estimate of the project is often observed in the dataset of public procurement auctions in Japan (e.g., Asai et al., 2018) and on highway procurement auctions in the US (e.g., Lewis and Bajari, 2011), it is unobservable in our dataset. Because of this constraint, we cannot directly normalize bids by the cost estimate of the project.

submitted bids while controlling for auction fixed effects. Then, we compute the gap between identically submitted and other bids while removing auction-specific effects. Table VI reports the regression results. We find identically submitted bids are on average around S\$30,000 lower than others. For all the specifications, our estimates are statistically significant at 5% level.

[Table VI Around Here]

To explain identically submitted (collusive) bids are lower than other (competitive) bids, we consider two possibilities based on the conjecture that bids are positively correlated with costs as follows:

(a)   Every firm is ex-ante symmetric. The most efficient member among a cartel determines a bid as if it competes against non-cartel rivals.

(b)   Collusive firms that submit identical bids are more efficient than non-cartel rivals. A randomly selected member among a cartel determines a bid as if it competes against non-cartel rivals.

In either case, we need to assume two types of bids are observed in our sample auctions: competitive bids (type 1) and collusive bids (type 2).

Considering the presence of auction heterogeneity and asymmetric bid distributions, we use the framework by Krasnokutskaya (2011). To use this approach that focuses on first-price sealed-bid auctions in which two types of bid distributions are observed, we restrict our focus on auctions in which (1) lowest bidders win, which account for around half of our samples, and (2) at most two firms submit identical bids, which account for around 90% of our samples. The second restriction is required, given that collusive bidding strategy is likely to depend on the size of a cartel. Then, we impose further assumptions as below.

Assumption. *To utilize her approach, we further impose the following:*

1. *A procurer allocates the project to the lowest bidder and randomly allocates the project if there are multiple lowest bidders.*

2. *The submission of identical bids is not accidental.*

3. *An auction participant knows the set of rival firms and, in particular, the group of firms that submit identical bids (cartel), when planning a bid.*

4. *An auction participant independently draws the cost, conditional on unobserved auction characteristics.*

5. *Either one of the following is satisfied.*

   (a) *Type 1's cost distribution is identical to type 2's. The cartel plans a bid based on the minimum of members' costs as if the most efficient member competitively bids against non-cartel rivals.*

   (b) *Type 1's cost distribution is different from type 2's. The cartel plans a bid based on the cost randomly selected from members' costs as if a randomly selected member competitively bids against non-cartel rivals.*

Before discussing the actual procedures of structural analysis, we briefly comment on each assumption. The former part of Assumption 1 requires all firms know auctions in which lowest bidders win ex ante. To see how reasonable this part is, we predict when lowest bidders win contracts only based on the information available to us. Considering our sample firms are likely to have more information than ours, our estimates are considered the lower bound of predictability for them. Using random forest model, we achieve an area under a receiver operating characteristic curve (AUC) of around 0.7 as reported in Appendix B. The latter part of Assumption 1 specifies allocation rule when identically submitted bids are lowest. As McAfee and McMillan (1992) suggest randomization rationalizes submitting identical bids as the optimal collusion mechanism, we consider a project is randomly allocated to one of the firms that submit lowest bids. Assumption 2 enables us to distinguish ineffective

bids from effective bids that are used to recover cost distributions. Assumptions 3 and 4 allow us to recover cost distributions under the framework that allows auction heterogeneity and asymmetric bid distributions. Assumption 5 allows us to predict bidding outcomes and winners' costs in our simulations. Scenario (a) corresponds to the first possibility above (a) and resembles the assumption used by Li and Zhang (2015). Scenario (b) encompasses the second possibility above (b).

First, we non-parametrically identify model primitives using subsample auctions we select based on the procedure described in Appendix C.3. Under Krasnokutskaya (2011), the cost to the type-i bid is characterized as the product of a common component Y that is known to all firms and an individual component c^i that is privately observable, that is, $Y \times c^i$. Under this condition, the equilibrium-bid function takes a special functional form. In particular, for type i, the equilibrium-bid function is

$$B^i \;=\; Y \times \sigma_i(c^i), \tag{4}$$

where $\sigma_i(.)$ denotes the equilibrium-bid function for type i where Y is 1. Let b^i be the corresponding equilibrium bid where Y is 1, that is, $b^i = \sigma_i(c^i)$.

Paarsch and Hong (2006) suggest:

$$b^i \;=\; c^i + \frac{(1 - G_0^i(b^i))(1 - G_0^{-i}(b^i))}{N_{-i}^E g_0^{-i}(b^i)(1 - G_0^i(b^i)) + (N_i^E - 1)g_0^i(b^i)(1 - G_0^{-i}(b^i))}, \tag{5}$$

where N_i^E (N_{-i}^E) is the number of effective bids for type i (the rival type of i), G_0^i (G_0^{-i}) is the CDF for the equilibrium-bid distribution of type i (the rival type of i) where Y is 1, and g_0^i (g_0^{-i}) is the corresponding PDF for G_0^i (G_0^{-i}). We consider at most two firms submit identical bids, so $N_2^E \in \{0, 1\}$.

Unfortunately, the presence of unobserved heterogeneity Y means G_0^i is unobservable to us. Instead, only the joint distribution of B^1 and B^2 is observable.

We start with removing Y from B^1 and B^2 to identify g_0^1 and g_0^2. Denoting the joint-

characteristic function of $\ln(B^1)$ and $\ln(B^2)$ by $C(.,.)$, $C(.,.)$ is

$$C(\tau_1, \tau_2) \;=\; E\left\{\exp\left[i\tau_1 \ln(B^1) + i\tau_2 \ln(B^2)\right]\right\}, \tag{6}$$

where i denotes the imaginary number $\sqrt{-1}$. Then, the deconvolution method allows us to derive the characteristic function of $\ln(Y)$, $\ln(b^1)$, and $\ln(b^2)$ by

$$
\begin{aligned}
C_{\ln(Y)}(\tau) &= \exp\left[\int_0^\tau C_1(0,u_2)/C(0,u_2)du_2 - iuE\{\ln(b^1)\}\right], \\
C_{\ln(b^1)}(\tau) &= C(\tau,0)/C_{\ln(Y)}(\tau), \\
C_{\ln(b^2)}(\tau) &= C(0,\tau)/C_{\ln(Y)}(\tau),
\end{aligned}
\tag{7}
$$

where $C_1(.,.)$ is the partial derivative of $C(.,.)$ with respect to the first argument. Without loss of generality, we normalize $E\{\ln(b^1)\}$ to zero. Then, the PDF of $\ln(Y)$, $\ln(b^1)$, and $\ln(b^2)$ is recovered by

$$
\begin{aligned}
f_{\ln(Y)}(x) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-i\tau x)C_{\ln(Y)}(\tau)d\tau, \\
f_{\ln(b^i)}(x) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-i\tau x)C_{\ln(b^i)}(\tau)d\tau.
\end{aligned}
\tag{8}
$$

Through a change of variable formula, we obtain:

$$
\begin{aligned}
f_Y(y) &= \frac{1}{y}f_{\ln(Y)}(\ln(y)), \\
g_0^i(b^i) &= \frac{1}{b^i}f_{\ln(b^i)}(\ln(b^i)).
\end{aligned}
\tag{9}
$$

Hence, f_Y, g_0^1, and g_0^2 are identified from the joint distribution of B^1 and B^2. F_Y, G_0^1, and G_0^2 are constructed from f_Y, g_0^1, and g_0^2. Given g_0^1, g_0^2, G_0^1, G_0^2, N_1^E, and N_2^E, equation (5) allows us to identify the PDFs and CDFs of the distributions for c^1 and c^2: f_0^1, f_0^2, F_0^1, and F_0^2. We follow the method of non-parametric estimation developed by Hickman and Hubbard (2015) that employs boundary-correction techniques. Estimated distributions of g_0^i, f_0^i, G_0^i, and F_0^i

are denoted by $\hat{g}_0^i$, $\hat{f}_0^i$, $\hat{G}_0^i$, and $\hat{F}_0^i$.[28]

We then test whether the two types of cost distributions are distinct or not. We first find the mean of type-2 cost distribution ($\hat{F}_0^2$) is lower than that of type-1 cost distribution ($\hat{F}_0^1$). We can reject the null that the difference in mean is zero at a two-tailed significance level of 5%. This finding is consistent with both scenarios of Assumption 5 because it supports the conjecture that a cartel is likely to plan its bid based on the most efficient member's cost or a cartel member is more efficient than a non-cartel rival. Moreover, we cannot reject the null that the difference in mean is zero at a two-tailed significance level of 5%, when we compare type-2 cost distribution and the first-order statistic of type-1 cost distribution. This finding is consistent with Scenario (a). Overall, our estimates of cost distributions are consistent with both scenarios of Assumption 5.[29]

Second, following Hubbard and Paarsch (2014), we compute the inverse equilibrium-bid function as the solution of the system of ordinary differential equations for auction j:

$$
\begin{bmatrix} \frac{d\sigma_1^{-1}(b)}{db} \\ \frac{d\sigma_2^{-1}(b)}{db} \end{bmatrix} = \begin{bmatrix} \frac{1-\hat{F}_0^1(\sigma_1^{-1}(b))}{\left(N_{j,1}^E+N_{j,2}^E-1\right)\hat{f}_0^1(\sigma_1^{-1}(b))} & \frac{N_{j,2}^E(\sigma_2^{-1}(b)-\sigma_1^{-1}(b))+b-\sigma_2^{-1}(b)}{(b-\sigma_2^{-1}(b))(b-\sigma_1^{-1}(b))} \\ \frac{1-\hat{F}_0^2(\sigma_2^{-1}(b))}{\left(N_{j,2}^E+N_{j,1}^E-1\right)\hat{f}_0^2(\sigma_2^{-1}(b))} & \frac{N_{j,1}^E(\sigma_1^{-1}(b)-\sigma_2^{-1}(b))+b-\sigma_1^{-1}(b)}{(b-\sigma_1^{-1}(b))(b-\sigma_2^{-1}(b))} \end{bmatrix}, \tag{10}
$$

with the initial and boundary value conditions: $\sigma_1^{-1}(\bar{c}) = \sigma_2^{-1}(\bar{c}) = \bar{c}$ and $\sigma_1^{-1}(\underline{b}) = \sigma_2^{-1}(\underline{b}) = \underline{c}$, where $\underline{c}$ ($\bar{c}$) denotes the estimate for the common[30] lower (upper) bound of the pseudo-cost distribution, $\underline{b}$ is the common lower bound for the equilibrium bid, and $N_{j,i}^E$ denotes the number of effective bids for type i. We solve the system using the numerical method developed by Bajari (2001). In particular, we first use his shooting algorithm (the "first" method of Bajari (2001)) to acquire the initial proposal for the subsequent routine, and then utilize his projection algorithm based on polynomial approximation (the "third" method of

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[28]We provide further detail of the estimation procedure in Appendix C.3. Figures A.VIII and A.IX in Appendix C presents the results of estimations.

[29]We provide further detail of the estimation procedure in Appendix D. Figure A.XI in Appendix D present the results of estimations.

[30]As in Bajari (2001) and Hubbard and Paarsch (2014), we assume that the support of the pseudo-cost distribution is identical across types.

Bajari (2001)) with the initial proposal we acquired in the previous step. We solve the system for every pair of $N_{j,1}^E$ and $N_{j,2}^E$, where $3 \leq N_{j,1}^E \leq 5$ and $0 \leq N_{j,2}^E \leq 2$.[31]

Third, we draw the effect of eliminating k^{th}-degree connections among the participating firms based on the estimate of equation (1). To account for the standard error of the estimated effect of one k^{th}-degree connection among the participating firms on the number of ineffective bids, we say the effect follows the distribution, $\tilde{\beta}^k \sim N\left[\hat{\beta}^k, SE(\beta^k)\right]$, where $\hat{\beta}^k$ is the estimate of β^k and $SE(\beta^k)$ is the standard error of the estimate of β^k in equation (1). We note, however, that the number of effective bids can be below one or even negative if $\tilde{\beta}^k$ is negative. To avoid this issue, we use the truncated effect, $\max\left\{\tilde{\beta}^k, 0\right\}$, instead of $\tilde{\beta}^k$. Although this process distorts the distribution of the estimate, its distortion is small, given that the probability of $\tilde{\beta}^k$ being negative is below 1% for every estimate used in our simulations. Given this adjustment, the effect of eliminating one k^{th}-degree connection among the firms participating in auction j follows $\max\left\{\tilde{\beta}^k, 0\right\}$. We define Δ_j as the effect of eliminating connections among the firms participating in the auction on the decrement in the number of ineffective bids in the auction. In simulations, however, the change in the number of ineffective bids has to be an integer, even if Δ_j is not. To resolve this problem, we also draw a random number y from $U(0,1)$ as a lottery to determine $\lceil \Delta_j \rceil$ or $\lfloor \Delta_j \rfloor$ for the actual reduction in the number of ineffective bids, where $\lceil . \rceil$ and $\lfloor . \rfloor$ designate ceiling and floor functions, respectively. If $y \leq \Delta_j - \lfloor \Delta_j \rfloor$ ($y > \Delta_j - \lfloor \Delta_j \rfloor$), the reduction in the number of ineffective bids, δ_j, is $\lceil \Delta_j \rceil$ ($\lfloor \Delta_j \rfloor$). It is trivial to prove the expected reduction in the number of ineffective bids becomes Δ_j.

Fourth, in correspondence with both scenarios of Assumption 5, we simulate bidding outcomes twice. Considering the median number of effective bids for auctions in which lowest bidders win and at most two firms submit identical bids is four, we assume there are three competitive firms and one cartel formed by two collusive firms that submit identical bids in a typical auction.

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[31]We provide further detail of the estimation procedure in Appendix C.3. As an example, figure A.X in Appendix C shows the inverse equilibrium-bid functions when $N_{j,1}^E = 4$ and $N_{j,2}^E = 1$.

In our first simulation, which corresponds to Scenario (a), we draw a cost from $\hat{F}_0^1$ for $N_{j,1}^E + N_{j,2}^E + N_j^I$ times, where $N_{j,1}^E = 3$, $N_{j,2}^E = 1$, and $N_j^I = 1$, reflecting Scenario (a). For the control case, we derive bids from the estimated equilibrium-bid function evaluated at $N_{j,1}^E = 3$ and $N_{j,2}^E = 1$. For the treatment case, when the reduced number of ineffective bids is one, a cartel is broken, which increases the number of competitive firms by two. On the other hand, when there is no effect on the number of ineffective bids, there would be no change in bidding outcomes between the control and counterfactual cases. In summary, we derive bids from the estimated equilibrium-bid function evaluated at $N_{j,1}^E = 3 + 2\min\{\delta_j, 1\}$ and $N_{j,2}^E = 1 - \min\{\delta_j, 1\}$. We note we use the minimum of two collusive firms' costs as the input when evaluating a type-2 bid if $N_j^I = 1$.

In our second simulation, which corresponds to Scenario (b), we draw a cost from $\hat{F}_0^i$ for $N_{j,i}^E$ times for each type i, where $N_{j,1}^E = 3$ and $N_{j,2}^E = 1$, in correspondence with Scenario (b). For ineffective bids, we draw a cost from $\hat{F}_0^2$ for N_j^I times, where $N_j^I = 1$. For the control case, we derive bids from the estimated inverse equilibrium-bid function evaluated at $N_{j,1}^E = 3$ and $N_{j,2}^E = 1$. For the treatment case, when the reduced number of ineffective bids is one, a cartel is broken, which increases the number of competitive firms by two. We note originally collusive firms have different technologies than originally competitive firms, although they share the same technologies in the previous simulation. On the other hand, when there is no effect on the number of ineffective bids, there would be no change in bidding outcomes between the control and counterfactual cases. In summary, we derive bids from the estimated equilibrium-bid function evaluated at $N_{j,1}^E = 3$ and $N_{j,2}^E = 1 + \min\{\delta_j, 1\}$. We note we use randomly selected one of two collusive firms' costs as the input when evaluating a type-2 bid if $N_j^I = 1$. Otherwise, we use each firm's cost as the input when evaluating each type-2 bid.

We then compute the winner's cost for each case and scenario. If $N_j^I = 0$, the winning bid cannot be identical to any ineffective bid. Then, the lowest bidder's cost is the winner's cost. If $N_j^I = 1$, the lowest bidder's cost is the winner's cost when the lowest bid is type 1,

but the winner's cost is randomly drawn from collusive firms' costs when the lowest bid is type 2.

We repeat the third and fourth steps 200 times per scenario. For each trial, we compute the gap in the winning bid and the winner's cost for the control and treatment cases. To make each gap scaleless, we normalize it by the winning bid for the control case. Then, we take the mean for each scenario. Specifically, we compute

$$
\begin{aligned}
db^* &= \text{mean}\left(\{(b^*_{j,counterfactual} - b^*_{j,control})/b^*_{j,control}\}_{j=1}^{200}\right), \\
dc^* &= \text{mean}\left(\{(c^*_{j,counterfactual} - c^*_{j,control})/b^*_{j,control}\}_{j=1}^{200}\right), \quad (11)
\end{aligned}
$$

where $b^*_{j,s}$ is the winning bid and $c^*_{j,s}$ is the winner's cost for auction j and case s. Although we need to draw an auction-specific cost shock y_j from F_Y to fully replicate the data-generation process, this process is not required to compute the "normalized" gap in the winning bid and the winner's cost. To see why, suppose we compute the statistics, dB^* and dC^*, which correspond to db^* and dc^*, based on the cost distributions convoluted by y_j. Then,

$$
\begin{aligned}
dB^* &= \text{mean}\left(\{(y_j b^*_{j,counterfactual} - y_j b^*_{j,control})/y_j b^*_{j,control}\}_{j=1}^{200}\right) = db^*, \\
dC^* &= \text{mean}\left(\{(y_j c^*_{j,counterfactual} - y_j c^*_{j,control})/y_j b^*_{j,control}\}_{j=1}^{200}\right) = dc^*. \quad (12)
\end{aligned}
$$

This finding suggests our results do not depend on y_j. Finally, to evaluate how the welfare gain is split between the government and firms, we also decompose $-dc^*$ into the following components:

$$
\text{Welfare Gain} = \overbrace{\text{Government Savings}}^{-db^*} + \overbrace{\text{Firm Gains}}^{-dc^*+db^*}
$$

$$
= -db^* + (-dc^* + db^*) = -dc^*.
$$

We report db^*, dc^*, government savings, and firm gains for each scenario.

We conduct this analysis for the eliminations of two types of ownership connections. First, we consider the elimination of one second-degree current-shareholder connection. Second, we consider the elimination of one fourth-degree current-shareholder connection.

5.2 Results

Figure III presents the effects of removing one second-degree connection on auction outcomes. We find the point estimate of db^* is -0.0051 under Scenario (a) and -0.0763 under Scenario (b). The 95% confidence interval of estimated db^* is below zero under Scenario (b), whereas it does not hold under Scenario (a). This result implies removing one second-degree connection reduces the winning bid by 0.5% under Scenario (a) and 7.6% under Scenario (b). The estimated effect is statistically significant at 5% level only under Scenario (b). Because a decrease in the winning bid is interpreted as government savings, this result also means removing one second-degree connection improves government savings by 0.5% under Scenario (a) and 7.6% under Scenario (b). We next find the point estimate of dc^* is -0.0718 under Scenario (a) and -0.0767 under Scenario (b). The 95% confidence interval of estimated dc^* is below zero under both scenarios. This result implies removing one second-degree connection reduces the winner's cost by 7.2% of the winning bid under Scenario (a) and 7.7% of the winning bid under Scenario (b). The estimated effects are statistically significant at 5% level, regardless of scenario. Lastly, we find the point estimate of firm gains is substantive (0.0667) under Scenario (a) and almost zero under Scenario (b). The 95% confidence interval of estimated firm gains is above zero only under Scenario (a). This result implies removing one second-degree connection improves firms' surplus by 6.7% of the winning bid under Scenario (a), but does not affect firms' surplus so much under Scenario (b). The estimated effect is statistically significant at 5% level only under Scenario (a).

[Figure III Around Here]

Figure IV shows the effects of removing one fourth-degree connection on auction out-

33

comes. The estimated effects follow the similar pattern as before. We find the point estimate of db^* is -0.0040 under Scenario (a) and -0.0509 under Scenario (b). The 95% confidence interval of estimated db^* is below zero only under Scenario (b). This result implies removing one fourth-degree connection reduces the winning bid by 0.4% under Scenario (a) and 5.1% under Scenario (b). The estimated effect is statistically significant at 5% level only under Scenario (b). In other words, this result suggests removing one fourth-degree connection improves government savings by 0.4% under Scenario (a) and 5.1% under Scenario (b). Our point estimate of dc^* is -0.0459 under Scenario (a) and -0.0412 under Scenario (b). The 95% confidence interval of estimated dc^* is below zero under both scenarios. This result implies removing one fourth-degree connection reduces the winner's cost by 4.6% of the winning bid under Scenario (a) and 4.1% of the winning bid under Scenario (b). The estimated effects are statistically significant at 5% level under both scenarios. We finally find the point estimate of firm gains is positive (0.0419) under Scenario (a), but negative (-0.0097) under Scenario (b). The 95% confidence interval of estimated firm gains is above zero under Scenario (a). This result implies removing one fourth-degree connection improves firms' surplus by 4.2% of the winning bid under Scenario (a). The estimated effect is statistically significant at 5% level. On the other hand, the 95% confidence interval of estimated firm gains overlaps 0 under Scenario (b). This result implies removing one fourth-degree connection aggravates firms' surplus by 1.0% of the winning bid under Scenario (b), while the estimated effect is not statistical significant at 5% level.

[Figure IV Around Here]

Overall, we find the robust evidence that eliminating ownership connections improves contractors' cost efficiency, regardless of scenario. This result highlights the inefficient nature of identical bidding induced by ownership connections: randomizing project allocation independently of firm efficiency when multiple firms submit the lowest bid. On the other hand, the effect on the winning bid and government savings depends on scenario. Under Scenario (a), a cartel's bidding strategy is still aggressive because it refers the most efficient

member's cost when planning a bid. Knowing this strategy, competitive firms also submit aggressive bids. Consequently, breaking a cartel does not reduce the winning bid so much, because increasing competitive pressure is offset by less aggressive bidding strategy. However, under Scenario (b), a cartel's bidding strategy is not so aggressive because it randomly refers one of the members' costs when planning a bid. Because increasing competitive pressure is not offset by less aggressive bidding strategy, breaking a cartel substantively reduces the winning bid. We finally note that there is substitutional relationship between government savings and firm gains. On the one hand, under Scenario (a), government savings are relatively small whereas firm gains are relatively large. On the other hand, under Scenario (b), firm gains are almost zero or negative whereas government savings are relatively large.

6   Conclusion

This paper studies the impact of ownership connections on prices and efficiency in the product market. In particular, we document identical bidding is positively correlated with having a shared owner (second-degree connection) or the shared owner's owner (fourth-degree connection). We also find identical bidding driven by ownership connections raises contract price for an auction in which the lowest bidder wins. This result is consistent with firms with shared owners rigging bids in order to raise the price of a contract. Combining reduced-form and structural analyses, we finally estimate the effects of excluding an ownership connection on the winning bid, the the winner's cost, government savings, and firm gains.

The findings in this paper are particularly relevant for policymakers who seek to improve or maintain competition in public procurement auctions. In particular, our analysis suggests a relevant piece of the bidders' information is the ownership structure or beneficial shareholders of the firm. Finally, we note our research is only possible due to the Singapore government's open data policies.

Ben Charoenwong, Assistant Professor of Finance, National University of Singapore Business School.

Kentaro Asai, Lecturer (Assistant Professor of Finance), Australian National University College of Business and Economics.

Online Appendix

A Robustness Tests

In this section, we present the results from robustness tests.

Figure A.I. Falsification
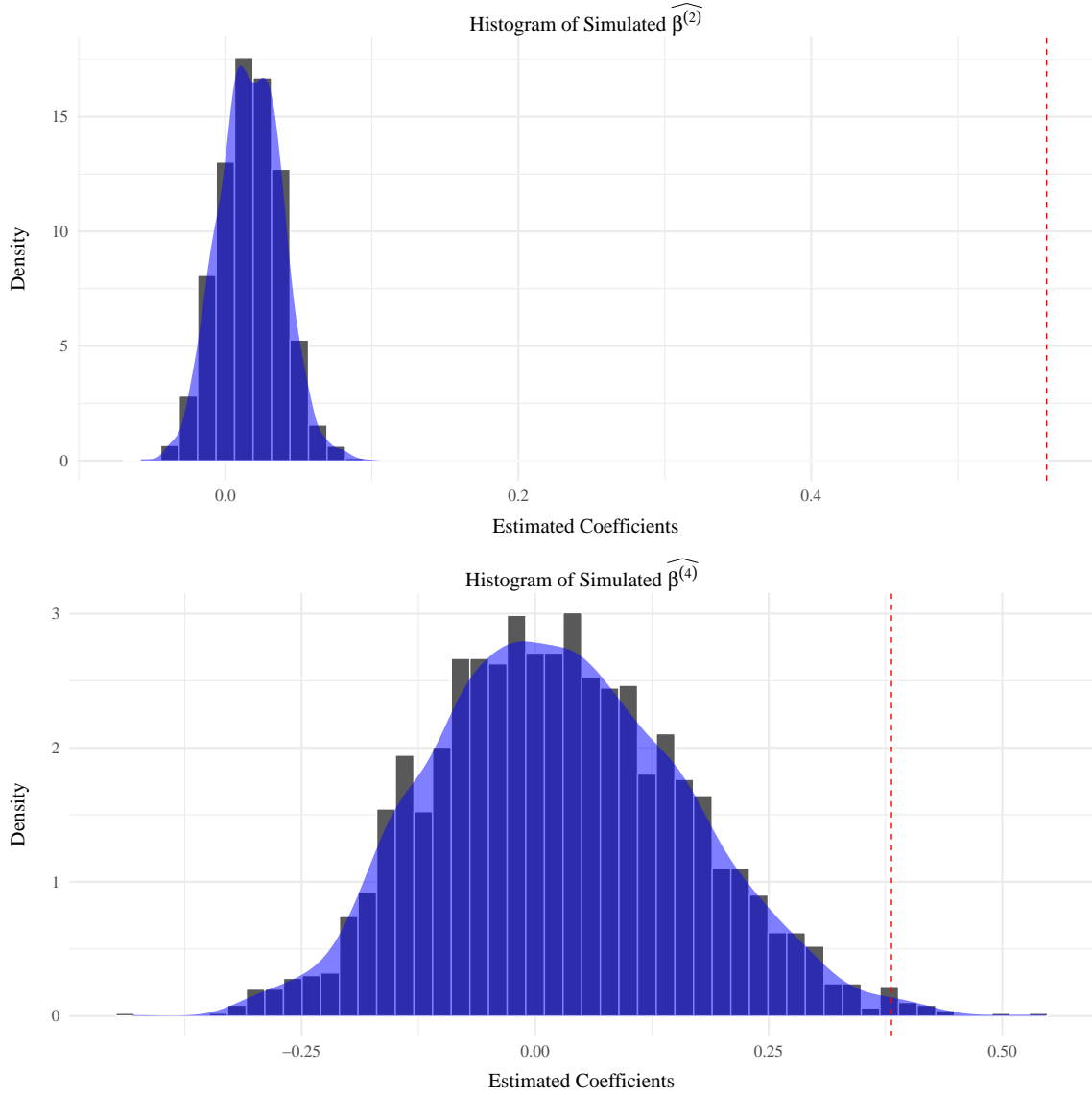
The figure above shows the histogram based on 2,500 simulated $\hat{\beta}^k$ for NC^k for $k = 2, 4$. Each simulation draws connections for every auction from the same procurer, auction category, and procurement-item type with replacement.

Figure A.II. Distribution of Bid Differences with Different Bidders

Number of observations: 3917 in (3,5]. 5134 in (5,10]. 1904 in (10,15]. 800 in (15,45]. Each histogram uses 50 bins.
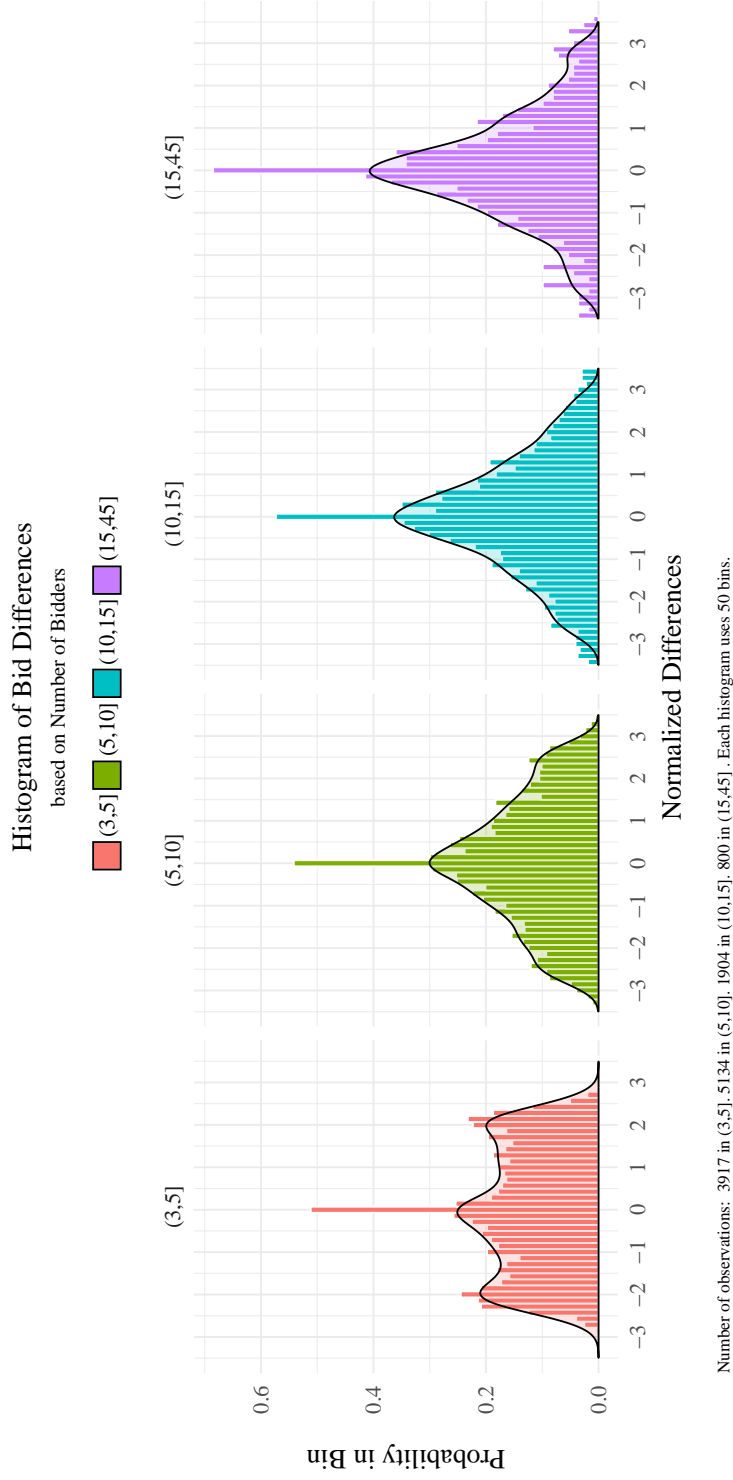
The figure above shows the histogram based on auctions with different numbers of participants. We see the mass in the bid-difference histograms still occurs at 0.

Figure A.III. Round Numbers in Bids

Distribution of Trailing 10–digits
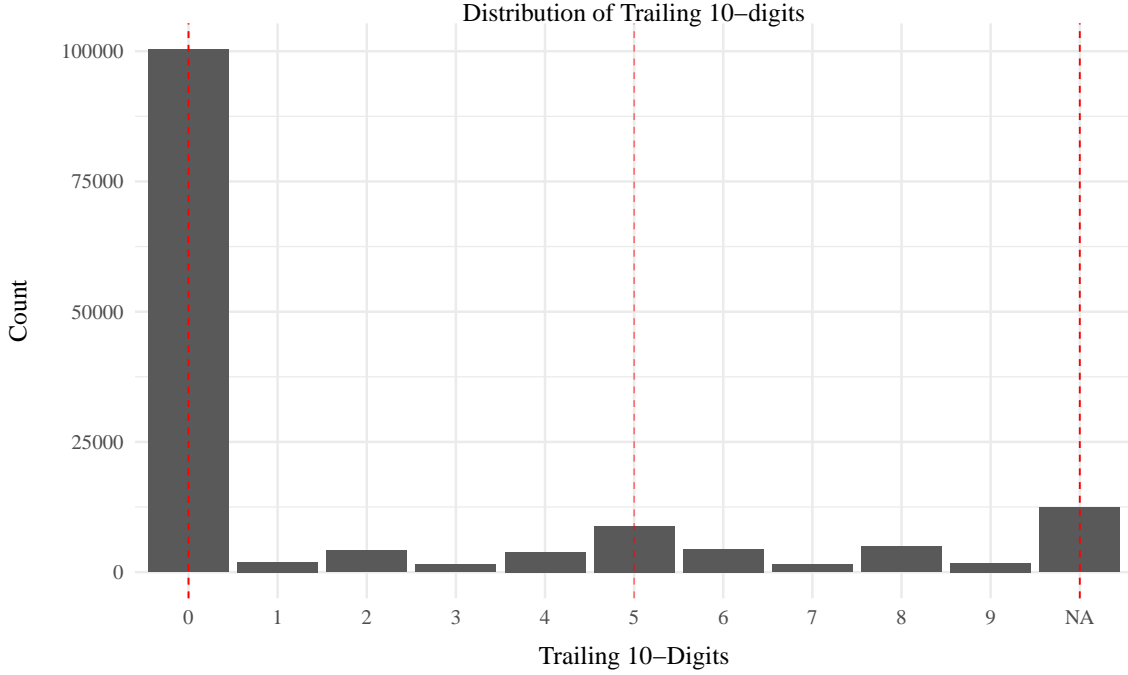
X (digits)	Pr (Bid Rounded to Nearest X | Bid > X)	Num. Bid > X
0	0.000	145,733
10	0.692	145,071
100	0.392	144,524
1,000	0.138	142,681
10,000	0.036	105,599
100,000	0.042	14,689
1,000,000	0.029	4,895

The figure above shows the histogram of trailing numbers in bids, shown for all bids. A 10-digits unit value of "NA" means the bid did not end in an integer. We find clustering at 0's and 5's. In addition, the table shows the fraction of bids greater than a particular value, the number of bids, and the probability that a bid appears rounded to that digit. For example, for digit = 10,000, the table shows the fraction of bids above S\$10,000, and counts the probability that a bid greater than S\$10,000 ends with "0,000".

B Predicting Government Preferences

In this section, we compare the different auction qualities that are associated with whether the lowest bid won the auction ex post. As the government reports, around 50% of the auctions have this "first price" property. This observation is relevant for our analysis for

Table A.I. Robustness: Number of Ineffective Bids and Ownership Networks

Dependent Variable:	Number of Ineffective Bids			
Fixed Effects		Num. Bidders	Num. Bidders & Procurer	Num. Bidders & Procurer×Category
	Panel A: All			
	(1)	(2)	(3)	(4)
NC^2	0.959***	0.675***	0.591***	0.557***
	(0.078)	(0.049)	(0.032)	(0.038)
NC^4	0.788***	0.725***	0.468***	0.386***
	(0.132)	(0.141)	(0.019)	(0.012)
Constant	0.330**			
	(0.135)			
Observations	22,098	22,098	22,098	22,098
R^2	0.142	0.270	0.325	0.377
	Panel B: Goods			
	(1)	(2)	(3)	(4)
NC^2	0.746***	0.598***	0.541***	0.524***
	(0.059)	(0.040)	(0.037)	(0.028)
NC^4	0.788***	0.719***	0.602***	0.351***
	(0.120)	(0.066)	(0.031)	(0.015)
Constant	0.169***			
	(0.049)			
Observations	7,880	7,880	7,880	7,880
R^2	0.078	0.246	0.268	0.375
	Panel C: Services			
	(1)	(2)	(3)	(4)
NC^2	0.970***	0.450***	0.380***	0.345***
	(0.084)	(0.122)	(0.099)	(0.105)
NC^4	0.705***	0.649***	0.401***	0.387***
	(0.154)	(0.160)	(0.019)	(0.012)
Constant	0.447***			
	(0.168)			
Observations	13,611	13,611	13,611	13,611
R^2	0.152	0.335	0.394	0.430

The table above shows the relation between the number of ownership connections in an auction and shareholder connections on the number of ineffective bids under the control of the different sets of fixed effects. Regressions are of the form $N_j^I = \sum_{k=2,4} \beta^k NC_j^k + f(Z_j) + f(R_j) + \epsilon_j$, where j indexes an auction and $f(Z_j)$ is the set of controls for auction-level characteristics, including different levels of fixed effects as specified in the "Fixed Effects" row. NC^k stands for connections of k degrees based on current-shareholder connections. Standard errors are clustered by procurer and shown in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A.II. Interaction of Bid Rounding with Ownership Connections

Dependent Variable:	Number of Ineffective Bids	
k-degree connection =	2	4
	(1)	(2)
NC^k	0.559***	0.371***
	(0.040)	(0.012)
$NC^k \times R_j$	-0.001	-0.187***
	(0.039)	(0.020)
Observations	22,098	22,098
R^2	0.377	0.342

The table above shows the relation between the number of ineffective bids and the number of ownership connections in an auction, interacted with the measure of bid rounding in an auction. Regressions are of the form $N_j^I = \beta^k NC_j^k + \gamma^k NC_j^k \times R_j + f(Z_j) + f(R_j) + \epsilon_j$, where j indexes an auction and $f(Z_j)$ is the set of controls for auction-level characteristics (Num. Bidders fixed effects and Procurer $\times$ Category fixed effects). NC^k stands for connections of k degrees based on current-shareholder connections. All regressions also include cubic controls of the average roundedness of bids in an auction, whose coefficients are suppressed for space. Standard errors are clustered by procurer and shown in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

us to understand the extent to which the government cares about the lowest bidder, and whether the preferences can be predicted by the data that we have collected. In the table, below, we report the average performance of three fitted models that are constructed to predict an auction's winner. However, because of limited data, we are unable to consider individual respondents or procurer-respondent fixed effects.

We also consider a more flexible empirical specification by using random forests, with 100 trees. We show that, given our dataset, a small number of trees is sufficient to generate stable results. We show the importance chart in Figure A.IV, based on either the mean decrease in classification accuracy or Gini impurity. According to both measures, the number of bidders, auction category, and whether other active auctions occur within a 30- or 10-day time frame are all important variables.

Overall, the more flexible random forests that permit more non-linearities in the data do not improve the overall performance too much, as shown in Figure A.V, which shows both the receiver operating characteristic (ROC) plot, which shows the performance in terms of the false positive rate and the true positive rate, and the overall performance summary

statistics. The best predictor would be a curve that is very steep initially then flat, taking the curvature towards the top-left corner. Overall, the random-forest model performs the best, with a correct rate of around 69%. The corresponding error rates are a 21% false-positive rate, and a 44% false-negative rate.

Figure A.IV. Random-Forest Models
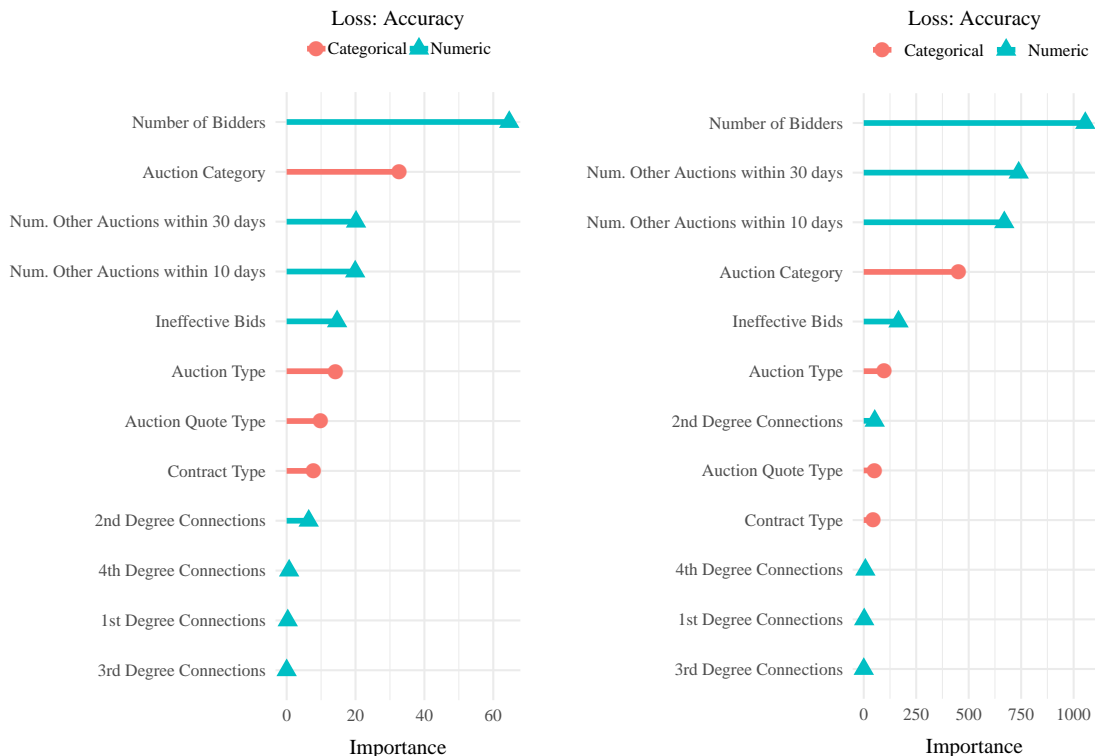
The figures above visualize the variables in the random forest based on a decreasing order of importance. We show the stability of the model based on both the mean decrease in the classification accuracy, defined as the probability of correct classifications, as well as the Gini impurity measure as the measure of importance, or loss function. Based on each choice, the model ranks the variables based on how much loss increases when a variable is dropped relative to the model that includes all variables. The variables are split into either categorical or numerical.

Table A.III. Linear Models for Lowest Bid Winning

Dependent variable:			Lowest Bid Wins		
Model		Linear		Probit	Logistic
	(1)	(2)	(3)	(4)	(5)
Number of Bidders	−0.117***	−0.119***	−0.120***	−0.320***	−0.544***
	(0.017)	(0.017)	(0.018)	(0.014)	(0.024)
Number of Bidders^2	0.006***	0.007***	0.007***	0.017***	0.030***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
Number of Bidders^3	−0.0001***	−0.0001***	−0.0001***	−0.0003***	−0.0005***
	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.0001)
Number of Other	−0.0001***	−0.0001***	−0.00001***	−0.0002***	−0.0003***
Auctions in 30 days	(0.00001)	(0.00001)	(0.00000)	(0.00003)	(0.0001)
Category = Goods		−0.067**	−0.012	−0.190**	−0.315**
		(0.028)	(0.032)	(0.081)	(0.132)
Category = Services		−0.050*	−0.006	−0.144*	−0.238*
		(0.029)	(0.029)	(0.080)	(0.131)
Type = Open Quotation		0.600***	0.545***	5.297	11.016
		(0.047)	(0.012)	(235.034)	(119.468)
Type = Open Tender		0.523***	0.423***	5.088	10.665
		(0.032)	(0.020)	(235.034)	(119.468)
Type = Construction				0.576***	0.953***
				(0.053)	(0.088)
Type = Facilities				0.551***	0.918***
				(0.051)	(0.085)
Type = Furniture				0.118**	0.195**
				(0.055)	(0.090)
Type = IT				0.249***	0.406***
				(0.040)	(0.065)
Type = Miscellaneous				−0.188***	−0.321***
				(0.068)	(0.112)
Type = Services				0.153***	0.262***
				(0.029)	(0.049)
Type= Transportation				0.155***	0.268***
				(0.049)	(0.081)
Type = Training				0.206*	0.334*
				(0.116)	(0.189)
Constant	0.916***			−4.098	−9.004
	(0.038)			(235.034)	(119.468)
Fixed Effects		Category	Procurer-Category		
Observations	16,582	16,582	16,582	16,582	16,582
R^2	0.121	0.139	0.219		

*Note:*   *p<0.1; **p<0.05; ***p<0.01

The table above presents the coefficients from a linear fixed-effects model, probit, and a logistic model to show what variables most correlate with whether the lowest bid in an auction won the auction. In the fixed-effects linear models, we cluster standard errors by procurer, consistent with our previous specifications.

Figure A.V. Performance of Predicted Lowest Price Winning

ROC Plot

— Random Forest ····· Probit — – Logit
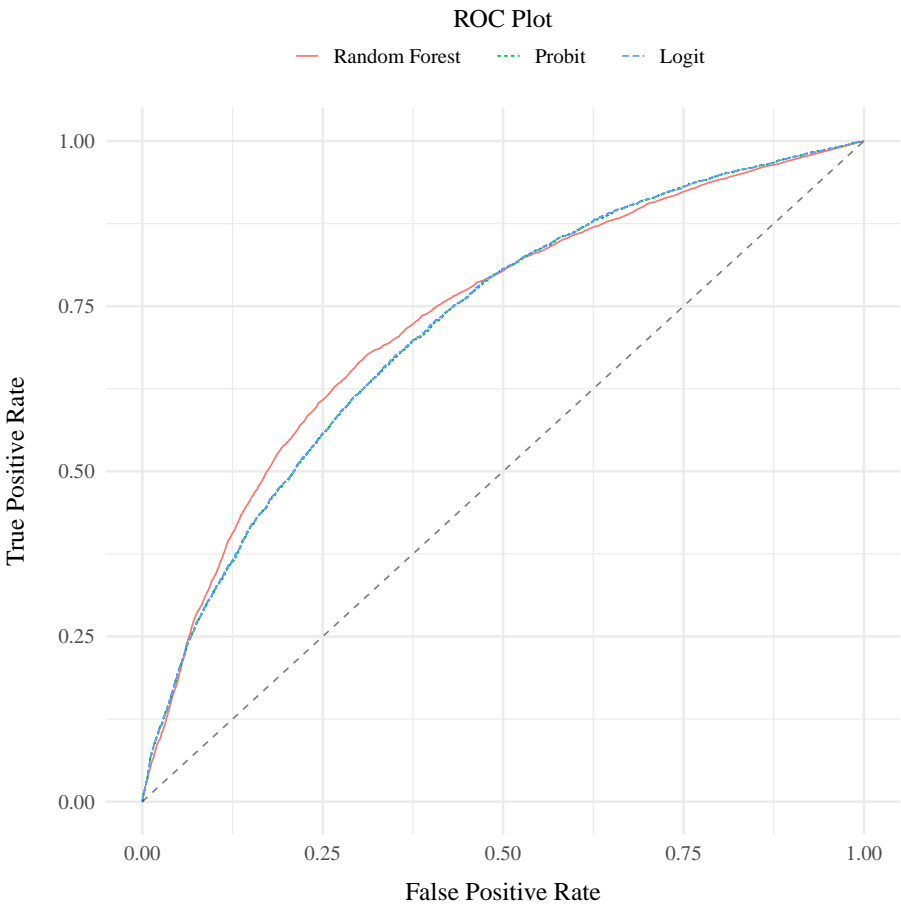
		Performance Metrics			
Model	Correct	Correct Positive	Correct Negative	False Positive	False Negative
Logit	0.671	0.498	0.791	0.209	0.502
Probit	0.671	0.500	0.790	0.210	0.500
Random Forest	0.694	0.558	0.789	0.211	0.442

Figure A.VI. Sensitivity to Number of Trees
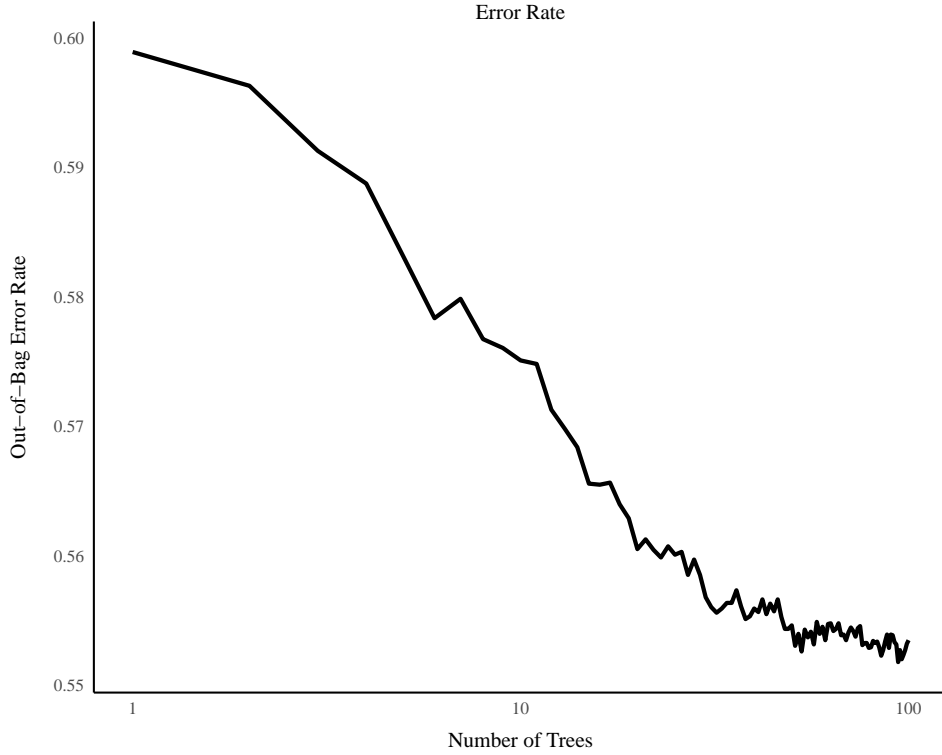
The figures above visualize the out-of-bag classification error rate for different numbers of trees in the model, ranging from one tree to 100 trees.

C   Detail of Structural Analysis

In this section, we describe the detail of structural analysis.

C.3   Non-parametric estimation of empirical distribution

For any non-parametric estimation of empirical distribution, we use the boundary-corrected kernel density estimator (KDE) developed by Hickman and Hubbard (2015). We choose a kernel, parameters, and bandwidths, following their approach.

C.3   Subsample Selection

To utilize Krasnokutskaya (2011), we use subsamples in our sample auctions for our structural estimation. Among our sample auctions in which lowest bidders win and at most two firms submit identical bids, we focus on auctions in which submitted values are moderate by excluding auctions in which the minimum bid is below S$10,000 or the maximum bid is above S$30,000. In addition, we restrict our focus on "Open Quotation" tenders to homogenize participation constraints across auctions in our subsamples. Then, we restrict to $N_1^E = 2$ and $N_2^E = 1$, leaving us 65 subsample auctions. The median winning bid for our subsample auctions is S$16,000, which almost matches the median winning bid for our sample auctions in which lowest bidders win. After this procedure, we exclude outliers from the sample as in Asker (2010). Figure A.VII shows outliers in the sample. After excluding outliers, our subsamples comprise 62 auctions.

C.3   Estimation Procedure

Our estimation procedure consists of the following steps:

1. $B_{m_i,l}^i$ denotes the m_i-th bid in auction l for type i that we observe from our bid data. The log transformation of bid data is performed to obtain $LB_{m_i,l}^i = \ln(B_{m_i,l}^i)$, $m_1 \in \{1, ..., N_1^E\}$, $m_2 \in \{1, ..., N_2^E\}$, for each auction l we use to recover the pseudo cost distributions. We rank $B_{m_i,l}^i$ such that $B_{x,l}^i < B_{x',l}^i$, where $x < x'$.

2. The joint-characteristic function of an arbitrary pair $(LB_{m_1,l}^1, LB_{m_2,l}^2)$ is estimated by

$$\hat{C}(\tau_1, \tau_2) = \frac{1}{N_1^E N_2^E} \sum_{1 \leq m_1 \leq N_1^E, 1 \leq m_2 \leq N_2^E} \frac{1}{L} \sum_{l=1}^L \exp(i\tau_1 LB_{m_1,l}^1 + i\tau_2 LB_{m_2,l}^2),$$

where L is the number of auctions we use for estimation. Then, we acquire the estimates of

characteristic functions as

$$
\begin{aligned}
\hat{C}_{\ln(Y)}(\tau) &= \exp\left[\int_0^\tau \hat{C}_1(0,u_2)/\hat{C}(0,u_2)du_2\right], \\
\hat{C}_{\ln(b^1)}(\tau) &= \hat{C}(\tau,0)/\hat{C}_{\ln(Y)}(\tau), \\
\hat{C}_{\ln(b^2)}(\tau) &= \hat{C}(0,\tau)/\hat{C}_{\ln(Y)}(\tau).
\end{aligned}
$$

3. The inversion formula is used to estimate densities $\hat{f}_{\ln(Y)}$, $\hat{f}_{\ln(b^i)}$, $i = 1, 2$, as

$$
\begin{aligned}
\hat{f}_{\ln(Y)}(u_y) &= \frac{1}{2\pi}\int_{-T_{\ln(Y)}}^{T_{\ln(Y)}} d_{T_{\ln(Y)}}(\tau)\exp(-i\tau u_y)\hat{C}_{\ln(Y)}(\tau)d\tau, \\
\hat{f}_{\ln(b^1)}(u_1) &= \frac{1}{2\pi}\int_{-T_{\ln(b^1)}}^{T_{\ln(b^1)}} d_{T_{\ln(b^1)}}(\tau)\exp(-i\tau u_1)\hat{C}_{\ln(b^1)}(\tau)d\tau, \\
\hat{f}_{\ln(b^2)}(u_2) &= \frac{1}{2\pi}\int_{-T_{\ln(b^2)}}^{T_{\ln(b^2)}} d_{T_{\ln(b^2)}}(\tau)\exp(-i\tau u_2)\hat{C}_{\ln(b^2)}(\tau)d\tau,
\end{aligned}
$$

where $u_y \in [\underline{LY}, \bar{LY}]$, $u_i \in [\underline{Lb}, \bar{Lb}]$, $i = 1, 2$, and $T_{\ln(Y)}$, $T_{\ln(b^1)}$, $T_{\ln(b^2)}$ are smoothing parameters. Following Krasnokutskaya (2011) , we introduce a damping factor $d_T(\tau)$ defined as $d_T(\tau) = \max\{1 - |\tau|/T, 0\}$ in the inversion formula. Through a change in the variable formula, we obtain

$$
\begin{aligned}
\hat{f}_Y(y) &= \frac{1}{y}\hat{f}_{\ln(Y)}(\ln(y)), \\
\hat{g}_0^i(b^i) &= \frac{1}{b^i}\hat{f}_{\ln(b^i)}(\ln(b^i)).
\end{aligned}
$$

Then, $\hat{G}_0^i$ is constructed from $\hat{g}_0^i$. We estimate the inverse equilibrium-bid function as

$$
\hat{\xi}_i(b^i) = b^i - \frac{(1 - \hat{G}_0^i(b^i))(1 - \hat{G}_0^{-i}(b^i))}{N_{-i}^E\hat{g}_0^{-i}(b^i)(1 - \hat{G}_0^i(b^i)) + (N_i^E - 1)\hat{g}_0^i(b^i)(1 - \hat{G}_0^{-i}(b^i))}.
$$

To implement the above estimation, we need to determine the smoothing parameters and the common support of bid distributions. As in Krasnokutskaya (2011) and Asker (2010), we choose the smoothing parameters and the common support of bid distributions based

on the moment-matching method. Because the mean of $\ln(b^1)$ is zero by normalization, the estimates for the means and variances of $\ln(Y)$, $\ln(b^1)$, and $\ln(b^2)$ are

$$\hat{\mu}(\ln(Y)) = \frac{1}{N_1^E L} \sum_{m_1=1}^{N_1^E} \sum_{l=1}^{L} LB_{m_1,l}^1,$$

$$\hat{v}(\ln(Y)) = \frac{1}{2(N_1^E L - 1)} \sum_{m_1=1}^{N_1^E} \sum_{l=1}^{L} \left[LB_{m_1,l}^1 - \frac{\sum_{m_1=1}^{N_1^E} \sum_{l=1}^{L} LB_{m_1,l}^1}{N_1^E L} \right]^2 +$$

$$\frac{1}{2(N_2^E L - 1)} \sum_{m_2=1}^{N_2^E} \sum_{l=1}^{L} \left[LB_{m_2,l}^2 - \frac{\sum_{m_2=1}^{N_2^E} \sum_{l=1}^{L} LB_{m_2,l}^2}{N_2^E L} \right]^2 -$$

$$\frac{1}{2(N_1^E N_2^E L - 1)} \times$$

$$\sum_{1 \le m_1 \le N_1^E, 1 \le m_2 \le N_2^E} \sum_{l=1}^{L} \left[\frac{\left(LB_{m_1,l}^1 - LB_{m_2,l}^2 \right) -}{\frac{\sum_{1 \le m_1 \le N_1^E, 1 \le m_2 \le N_2^E} \sum_{l=1}^{L} \left(LB_{m_1,l}^1 - LB_{m_2,l}^2 \right)}{N_1^E N_2^E L}} \right]^2,$$

$$\hat{v}(\ln(b^1)) = \frac{1}{N_1^E L - 1} \sum_{m_1=1}^{N_1^E} \sum_{l=1}^{L} \left[LB_{m_1,l}^1 - \frac{1}{N_1^E L} \sum_{m_1=1}^{N_1^E} \sum_{l=1}^{L} LB_{m_1,l}^1 \right]^2 - \hat{v}(\ln(Y)),$$

$$\hat{\mu}(\ln(b^2)) = \frac{1}{N_2^E L} \sum_{m_2=1}^{N_2^E} \sum_{l=1}^{L} LB_{m_2,l}^2 - \hat{\mu}(\ln(Y)),$$

$$\hat{v}(\ln(b^2)) = \frac{1}{N_2^E L - 1} \sum_{m_2=1}^{N_2^E} \sum_{l=1}^{L} \left[LB_{m_2,l}^2 - \frac{1}{N_2^E L} \sum_{m_2=1}^{N_2^E} \sum_{l=1}^{L} LB_{m_2,l}^2 \right]^2 - \hat{v}(\ln(Y)).$$

Our choices for the smoothing parameters and the common support of bid distributions are set to replicate these moments.

Moreover, we require the estimates of the inverse equilibrium-bid functions to be increasing in bids for both types.

Furthermore, we consider the estimated inverse equilibrium-bid functions to be inadmissible if estimated pseudo costs are negative.

Given these considerations, our choices for $\left\{ \underline{Lb}, \bar{Lb}, T_{\ln(b^1)}, T_{\ln(b^2)} \right\}$, $\left\{ \hat{\underline{Lb}}, \hat{\bar{Lb}}, \hat{T}_{\ln(b^1)}, \hat{T}_{\ln(b^2)} \right\}$, are defined as

$$\left\{\underline{\hat{Lb}}, \hat{\bar{Lb}}, \hat{T}_{\ln(b^1)}, \hat{T}_{\ln(b^2)}\right\}$$

$$\in \quad \arg\min_{\left\{\underline{Lb}, \bar{Lb}, T_{\ln(b^1)}, T_{\ln(b^2)}\right\}} \begin{bmatrix} \int_{\underline{Lb}}^{\bar{Lb}} u_1 \hat{f}_{\ln(b^1)}(u_1) du_1 - 0 \\ \int_{\underline{Lb}}^{\bar{Lb}} \left[u_1 - \int_{\underline{Lb}}^{\bar{Lb}} u_1 \hat{f}_{\ln(b^1)}(u_1) du_1 \right]^2 \hat{f}_{\ln(b^1)}(u_1) du_1 \\ -\hat{v}(\ln(b^1)) \\ \int_{\underline{Lb}}^{\bar{Lb}} u_2 \hat{f}_{\ln(b^2)}(u_2) du_2 - \hat{\mu}(\ln(b^2)) \\ \int_{\underline{Lb}}^{\bar{Lb}} \left[u_2 - \int_{\underline{Lb}}^{\bar{Lb}} u_2 \hat{f}_{\ln(b^2)}(u_2) du_2 \right]^2 \hat{f}_{\ln(b^2)}(u_2) du_2 \\ -\hat{v}(\ln(b^2)) \\ \int_{\underline{Lb}}^{\bar{Lb}} u_1 \hat{f}_{\ln(b^1)}(u_1) du_1 - 0 \\ \int_{\underline{Lb}}^{\bar{Lb}} \left[u_1 - \int_{\underline{Lb}}^{\bar{Lb}} u_1 \hat{f}_{\ln(b^1)}(u_1) du_1 \right]^2 \hat{f}_{\ln(b^1)}(u_1) du_1 \\ -\hat{v}(\ln(b^1)) \\ \int_{\underline{Lb}}^{\bar{Lb}} u_2 \hat{f}_{\ln(b^2)}(u_2) du_2 - \hat{\mu}(\ln(b^2)) \\ \int_{\underline{Lb}}^{\bar{Lb}} \left[u_2 - \int_{\underline{Lb}}^{\bar{Lb}} u_2 \hat{f}_{\ln(b^2)}(u_2) du_2 \right]^2 \hat{f}_{\ln(b^2)}(u_2) du_2 \\ -\hat{v}(\ln(b^2)) \end{bmatrix}^T,$$

s.t. $\quad \sum_{i=1}^{2} \sum_{n=0}^{K} \left[\mathbf{1}\left\{\hat{g}_0^i(t_n) < 0\right\} + \mathbf{1}\left\{\hat{\xi}_i(t_n) < 0\right\} + \mathbf{1}\left\{d\hat{\xi}_i(t_n) < 0\right\} \right] \leq 0,$

where we define the grid $\exp(\underline{Lb}) = t_0 < t_1 < ... < t_{K-1} < t_K = \exp(\bar{Lb})$ ($K = 100$ for our reported results). The objective function captures the gap between predicted and observed moments. The first term in the constraint represents the penalty against negative values of estimated densities for b^1 and b^2. The second term is the penalty against negative values of estimated pseudo costs whereas the third term represents the penalty against decreasing inverse equilibrium-bid functions. Given the estimate $\left[\underline{\hat{Lb}}, \hat{\bar{Lb}}\right]$, the consistent estimator for the support of $[\underline{LY}, \bar{LY}]$ becomes $\left[\min_{1 \leq l \leq L}\{LB_{1,l}^1\} - \underline{\hat{Lb}}, \max_{1 \leq l \leq L}\{LB_{2,l}^1\} - \hat{\bar{Lb}}\right]$. Let this interval be $\left[\underline{\hat{LY}}, \hat{\bar{LY}}\right]$.

Then, our choice for $T_{\ln(Y)}$, $\hat{T_{\ln(Y)}}$, is defined as

$$\hat{T}_{\ln(Y)}$$

$$\in \underset{T_{\ln(Y)}}{\arg\min} \left[\begin{array}{c} \int_{\underline{\hat{LY}}}^{\bar{\hat{LY}}} u_y \hat{f}_{\ln(Y)}(u_y) du_y - \hat{\mu}(\ln(Y)) \\ \int_{\underline{\hat{LY}}}^{\bar{\hat{LY}}} \left[u_y - \int_{\underline{\hat{LY}}}^{\bar{\hat{LY}}} u_y \hat{f}_{\ln(Y)}(u_y) du_y \right]^2 \hat{f}_{\ln(Y)}(u_y) du_y - \hat{v}(\ln(Y)) \\ \int_{\underline{\hat{LY}}}^{\bar{\hat{LY}}} u_y \hat{f}_{\ln(Y)}(u_y) du_y - \hat{\mu}(\ln(Y)) \\ \int_{\underline{\hat{LY}}}^{\bar{\hat{LY}}} \left[u_y - \int_{\underline{\hat{LY}}}^{\bar{\hat{LY}}} u_y \hat{f}_{\ln(Y)}(u_y) du_y \right]^2 \hat{f}_{\ln(Y)}(u_y) du_y - \hat{v}(\ln(Y)) \end{array} \right]^T ,$$

$$\text{s.t.} \qquad \sum_{n=0}^{K} \mathbf{1}\left\{ \hat{f}_Y(t_n) < 0 \right\} + \mathbf{1}\left\{ T_{\ln(Y)} \geq 50 \right\} \leq 0,$$

where we define the grid $\exp(\underline{\hat{LY}}) = t_0 < t_1 < \ldots < t_{K-1} < t_K = \exp(\bar{\hat{LY}})$ ($K = 100$ for our reported results). As in the previous optimization problem, the objective function captures the gap between predicted and observed moments. The first term in the constraint represents the penalty against negative value of the estimated density for $\ln(Y)$. The second term in the constraint represents the penalty against too large smoothing parameter that is likely to end up with the estimated PDF possessing a wavy tail.

Regarding the first optimization problem, we use $[-0.4, 0.4]$ for the initial value of $\left[\underline{Lb}, \bar{Lb} \right]$ and arbitrary one of $\{3, 5, 7, 9, 11\}$ for the initial value of $T_{\ln(b^i)}$. Regarding the second optimization problem, we use arbitrary one of $\{5, 10, 15, 20\}$ for the initial value of $T_{\ln(Y)}$. For each optimization, we first try every possible set of initial values and obtain local optima. We second choose the one that minimizes the objective function without violating the constraint.

From this analysis, we find $\left[\underline{\hat{LY}}, \bar{\hat{LY}} \right] = [9.71, 10.61]$, $\left[\underline{\hat{Lb}}, \bar{\hat{Lb}} \right] = [-0.45, 0.40]$, $\hat{T}_{\ln(Y)} = 18.83$, $\hat{T}_{\ln(b^1)} = 11.00$, and $\hat{T}_{\ln(b^2)} = 11.00$. Figure A.VIII shows the distributions of auction-specific and individual components of bid distributions based on this deconvolution process.

4. Using the estimated inverse equilibrium-bid function $\hat{\xi}_i(b^i)$, we estimate pseudo costs. Then, $\hat{f}^i_{0,raw}$ is the boundary-corrected KDE for the pseudo costs. $\hat{F}^i_{0,raw}$ is constructed from $\hat{f}^i_{0,raw}$. The support of $\hat{F}^i_{0,raw}$ is $\left[\hat{\xi}_i\left(\exp\left(\underline{\hat{Lb}}\right)\right), \hat{\xi}_i\left(\exp\left(\bar{\hat{Lb}}\right)\right) \right]$. However, considering

the pseudo cost distributions have the common support, we replace the support of each distribution by

$$[\underline{c}, \bar{c}] = \left[\max_{1 \le i \le 2} \left\{ \hat{\xi}_i \left(\exp \left(\underline{\hat{Lb}} \right) \right) \right\}, \min_{1 \le i \le 2} \left\{ \hat{\xi}_i \left(\exp \left(\hat{\bar{Lb}} \right) \right) \right\} \right].$$

Then, we adjust densities by $\hat{f}_0^i(c^i) = \hat{f}_{0,raw}^i(c^i)/\left(\hat{F}_{0,raw}^i(\bar{c}) - \hat{F}_{0,raw}^i(\underline{c}) \right)$. $\hat{F}_0^i$ is adjusted in accordance with $\hat{f}_0^i$. Figure A.IX shows the estimated pseudo cost distributions of type-1 and type-2 bids on $[\underline{c}, \bar{c}]$ before this adjustment.

C.3   Numerical Method for Asymmetric Auctions

We combine the "first" and "third" methods of Bajari (2001) to solve equation (10). First, we use his shooting algorithm to obtain the initial proposals for the subsequent estimations. Let $\{s_1(b; \underline{b}), s_2(b; \underline{b})\}$ be the solution of the system where $\sigma_1^{-1}(\underline{b}) = \sigma_2^{-1}(\underline{b}) = \underline{c}$. Then, the shooting algorithm consists of the following steps:

1. Fix $b_{low} = \underline{c}$ and $b_{high} = \bar{c}$.

2. Set $b_{guess} = \frac{1}{2}(b_{low} + b_{high})$.

3. Determine whether the system $\{s_1(b; b_{guess}), s_2(b; b_{guess})\}$ diverges, that is, whether it is in S^2, where $S = \{s : \ s$ is C^1, $s : [\underline{c}, \bar{c}] \to [\underline{c}, \bar{c}]$ and $s(b) < b$ for all $b < \bar{c}\}$.

4. If $\{s_1(b; b_{guess}), s_2(b; b_{guess})\}$ is in S^2, set $b_{high} = b_{guess}$.

5. If $\{s_1(b; b_{guess}), s_2(b; b_{guess})\}$ is not in S^2, set $b_{low} = b_{guess}$.

6. If $b_{high} - b_{low} < \epsilon$, stop. Otherwise, go to step 2.

7. After the stop, set $b_{min} = b_{high}$ and $b_0 = \frac{1}{2}(b_{low} + b_{high})$.

Although Bajari (2001) proves $\{s_1(b; b_{min}), s_2(b; b_{min})\}$ converges to the solution of the system as $\epsilon \to 0$, this shooting mechanism is inherently unstable, and this instability cannot be eliminated by changing the numerical methodology of the solver (Fibich and Gavish, 2011). In

particular, this instability becomes severe when the number of effective bids in the auction is large. Indeed, $\{s_1(b; b_{min}), s_2(b; b_{min})\}$ deviates from $\bar{c}$ as $b \to \bar{c}$ when the number of effective bids is large, as presented in the left panel of Figure A.X. We therefore fix this problem using a completely different approach. For this purpose, we use the projection algorithm based on polynomial approximation. Specifically, we approximate the inverse equilibrium-bid function by the polynomial of degree 4. The approximated inverse equilibrium-bid function is

$$\hat{\sigma}_i^{-1}(b; \alpha, \underline{b}) = \sum_{k=1}^{4} \alpha_{i,k}(b - \underline{b})^k + \underline{c},$$

where $\alpha = \{\alpha_{i,k}\}_{1 \leq i \leq 2, 1 \leq k \leq 4}$. Then, the projection algorithm consists of the following steps.

1. Acquire the coefficients on $b - b_0$ for a polynomial of degree 4 that is a best fit for $s_i(b; b_{min}) - \underline{c}$ for each i. In this approximation, we use the closed interval on which $\frac{ds_i(b; b_{min})}{db} \geq 0$ and $s_i(b; b_{min}) \leq b$ for all b on the interval. The interval starts at b_{start}, where $b_{start} = b_0$ or $s_i(b_{start} - \epsilon; b_{min}) > b_{start} - \epsilon$ or $\frac{ds_i(b_{start} - \epsilon; b_{min})}{db} < 0$ for any $0 < \epsilon < \bar{\epsilon}$ ($\bar{\epsilon}$ is a certain threshold), and ends at b_{end}, where $b_{end} = \bar{c}$ or $s_i(b_{end} + \epsilon; b_{min}) > b_{end} + \epsilon$ or $\frac{ds_i(b_{end} + \epsilon; b_{min})}{db} < 0$ for any $0 < \epsilon < \bar{\epsilon}$ ($\bar{\epsilon}$ is a certain threshold). If multiple such intervals exist, we use the one that starts from the smallest b_{start}. From this approximation. we obtain the initial proposal $\{\alpha_0, b_0\}$.

2. We define the grid $\underline{b} = t_0 < t_1 < ... < t_{K-1} < t_K = \bar{c}$ ($K = 50$ for our reported results). Using $\{\alpha_0, b_0\}$ as the initial proposal, we solve the estimates of $\{\alpha, \underline{b}\}$, $\{\hat{\alpha}, \hat{\underline{b}}\}$, defined as

$$\{\hat{\alpha}, \underline{\hat{b}}\}$$

$$\in \underset{\{\alpha, \underline{b}\}}{\arg\min} \left\{ \sum_{n=0}^{K} \left[\left[\begin{array}{c} \frac{d\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b})}{db} \\ \frac{d\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b})}{db} \end{array} \right] - \left[\begin{array}{c} \frac{1-\hat{F}_0^1(\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))}{(N_1^E+N_2^E-1)\hat{f}_0^1(\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))} \times \\ \frac{N_2^E(\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b})-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))+t_n-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b})}{(t_n-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))(b-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))} \\ \frac{1-\hat{F}_0^2(\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))}{(N_2^E+N_1^E-1)\hat{f}_0^2(\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))} \times \\ \frac{N_1^E(\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b})-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))+t_n-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b})}{(t_n-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))(t_n-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))} \end{array} \right] \right]^T \right.$$

$$\left[\left[\begin{array}{c} \frac{d\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b})}{db} \\ \frac{d\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b})}{db} \end{array} \right] - \left[\begin{array}{c} \frac{1-\hat{F}_0^1(\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))}{(N_1^E+N_2^E-1)\hat{f}_0^1(\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))} \times \\ \frac{N_2^E(\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b})-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))+t_n-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b})}{(t_n-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))(t_n-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))} \\ \frac{1-\hat{F}_0^2(\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))}{(N_2^E+N_1^E-1)\hat{f}_0^2(\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))} \times \\ \frac{N_1^E(\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b})-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))+t_n-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b})}{(t_n-\hat{\sigma}_1^{-1}(t_n;\alpha,\underline{b}))(t_n-\hat{\sigma}_2^{-1}(t_n;\alpha,\underline{b}))} \end{array} \right] \right]$$

$$\left. + P \sum_{i=1}^{2}(\hat{\sigma}_i^{-1}(\bar{c};\alpha,\underline{b})-\bar{c})^2 \right\}.$$

We choose sufficiently large P such that the estimated inverse equilibrium-bid function for each type converges enough to $\bar{c}$ as $b \to \bar{c}$ ($P = 5,000$ for our reported results). $\{\hat{\sigma}_1^{-1}(b;\hat{\alpha},\underline{\hat{b}}), \hat{\sigma}_2^{-1}(b;\hat{\alpha},\underline{\hat{b}})\}$ is the final estimate for the inverse equilibrium-bid function. We estimate the inverse equilibrium-bid functions for every pair of N_1^E and N_2^E, where $3 \leq N_1^E \leq 5$ and $0 \leq N_2^E \leq 2$.

The right panel of Figure A.X shows the final estimates of the inverse equilibrium-bid functions converge enough to $\bar{c}$, even when the number of effective bids in the auction is relatively large. This finding suggests the instability problem of the shooting algorithm is fixed by the projection algorithm based on polynomial approximation.

C.3   Confidence Intervals

To acquire confidence intervals, we use bootstrap approach. We construct a resample by randomly selecting auctions from the original subsample with replacement with size equal to the number of auctions in the original subsample. Then, we estimate $\hat{f}_Y, \hat{g}_0^i, \hat{f}_0^i, \hat{\sigma}_i^{-1}(b;\hat{\alpha},\underline{\hat{b}}), \forall i,$

using each resample. We repeat this process 500 times.

To reduce computational burdens, we keep using the same smoothing parameters and bid bounds $(\underline{\hat{Lb}}, \hat{\bar{Lb}}, \hat{T}_{\ln(b^1)}, \hat{T}_{\ln(b^2)}, \underline{\hat{LY}}, \hat{\bar{LY}}, \hat{T}_{\ln(Y)})$ for the estimates of $\hat{f}_Y, \hat{g}_0^i, \hat{f}_0^i, \forall i$.

To further reduce computational burdens, we skip the first step and start from the second step using the the initial value used for the original subsample $\{\alpha_0, b_0\}$ for the estimates of $\hat{\sigma}_i^{-1}(b; \hat{\alpha}, \underline{\hat{b}}), \forall i$. If there still remain resamples from which the minimized value of the objective function, less the penalty term against the deviation from the terminal condition, exceeds 100, we use the initial value $\{\alpha_0, b_{j+1}\}$ for these resamples, where $b_{j+1} = b_j + 0.05$. We repeat this process three times.

Given $\hat{f}_Y, \hat{g}_0^i, \hat{f}_0^i, \hat{\sigma}_i^{-1}(b; \hat{\alpha}, \underline{\hat{b}}), \forall i$, we compute test statistics for each resample. Obtaining the distributions of test statistics, we are able to construct their confidence intervals.

Figure A.VII. Outliers in the Subsample

The left (right) plot is the scatter plot of log type-1 bids that are largest (smallest) among the two bids and log type-2 bids. The outliers are represented by red circles.
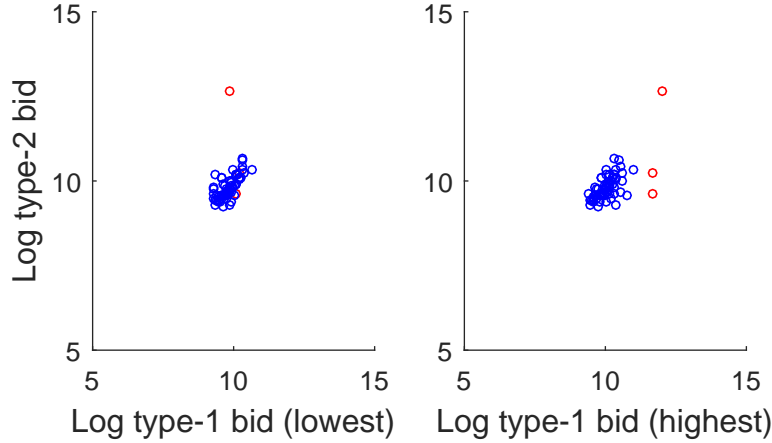
Figure A.VIII. Auction and Individual Components of Bid Distributions

The top panel plots $\hat{f}_Y$. The bottom left panel plots $\hat{g}_0^1$, whereas the bottom right panel plots $\hat{g}_0^2$. The solid lines depict the point estimates of the PDFs. The dotted lines show 5% and 95% point-wise quantiles of the estimated distributions.
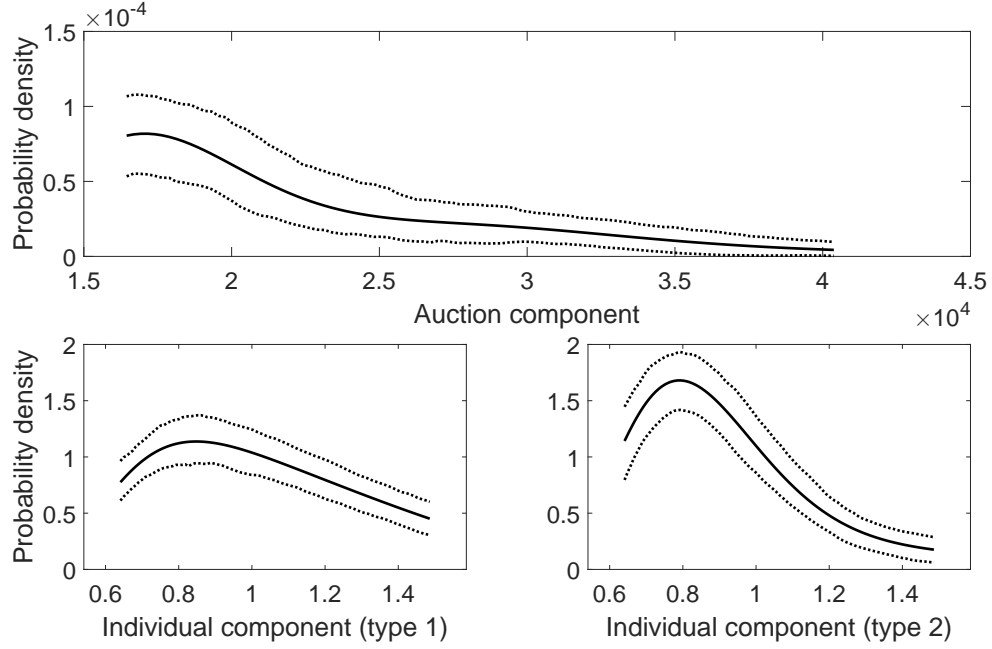
Figure A.IX. Estimated Cost Distributions

The left panel plots $\hat{f}^1_{0,raw}$, whereas the right panel plots $\hat{f}^2_{0,raw}$. The solid lines depict the point estimates of the PDFs. The dotted lines show 5% and 95% point-wise quantiles of the estimated distributions.
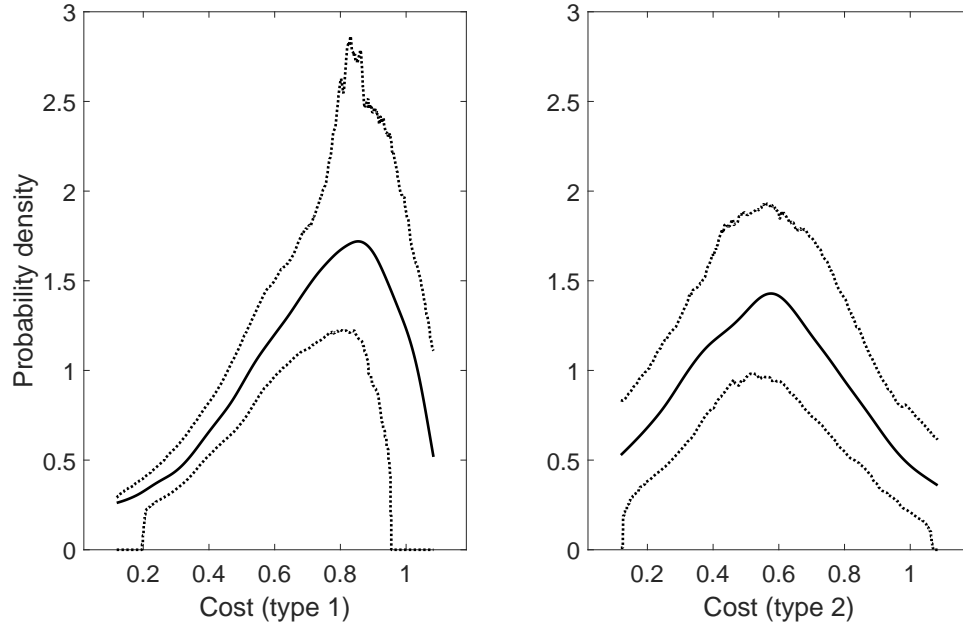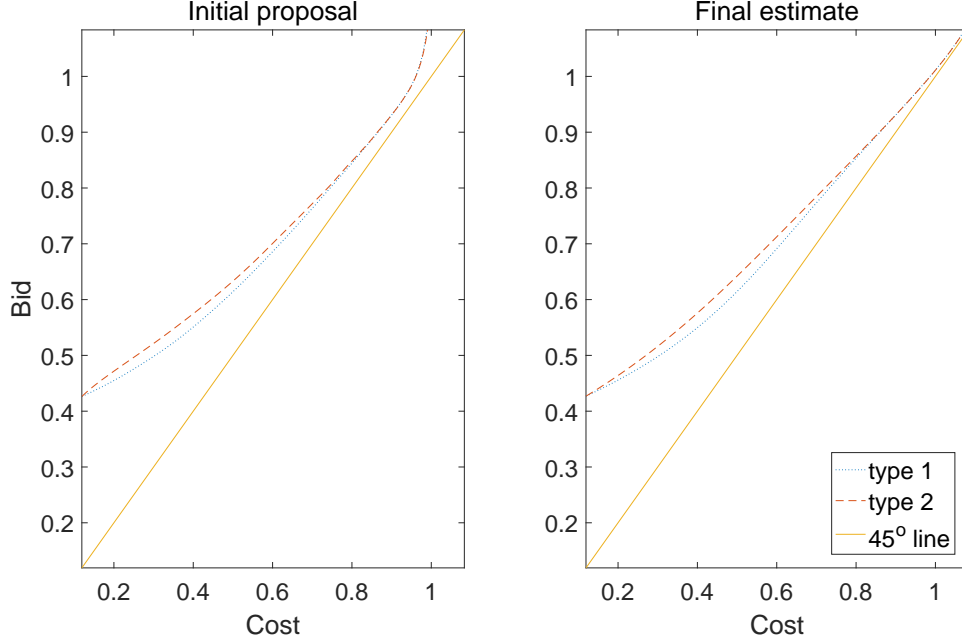
Figure A.X. Inverse Equilibrium-Bid Functions

The left panel plots the initial proposals for the inverse equilibrium-bid functions, whereas the right panel plots the final estimates for them ($N_1^E = 4$ and $N_2^E = 1$).

# D   Asymmetry of Competitive and Collusive Bids

In this section, we compare the distributions of pseudo costs for type-1 and type-2 bids. The upper middle panel of Figure A.XI presents the estimated PDFs of pseudo costs for type-1 and type-2 bids. The upper left panel shows the estimated distribution of first-order statistic of pseudo cost for type 1, whereas the upper right panel shows that of second-order statistic. We find the estimated distribution of first-order statistic of pseudo cost for type 1 is relatively similar to the estimated distribution of pseudo cost for type 2. On the other hand, the other type-1-related distributions are substantively different from the estimated distribution of pseudo cost for type 2.

To further investigate the similarity between type-2 pseudo cost and first-order statistic of type-1 pseudo cost, we compute the difference of mean type-2 distribution and mean type-

1-related distribution in the bottom panel of the figure. We find the gap in mean is estimated as 0.06 when we compare to first-order statistic of type-1 pseudo cost, whereas it is estimated as 0.17 and 0.29 when we compare to type-1 pseudo cost and second-order statistic of type-1 pseudo cost, respectively. Using bootstrap, we also construct a 95% confidence interval for each difference in mean. We cannot reject the null that the difference in mean is zero at a two-tailed significance level of 5%, when we compare to first-order statistic of type-1 pseudo cost. However, we reject the null, when we compare to type-1 pseudo cost and second-order statistic of type-1 pseudo cost.

Figure A.XI. Comparison of Pseudo Cost Distributions

In all the panels in the upper part of the figure, we represent the estimated PDF of type-2 pseudo cost via connected line, whereas we show the estimated PDF of type-1-related pseudo cost distribution via dashed line. In the upper left panel, we show the estimated distribution of first-order statistic of pseudo cost for type 1. In the upper middle panel, we show the estimated distribution of pseudo cost for type 1. In the upper right panel, we show the the estimated distribution of second-order statistic of pseudo cost for type 1. Corresponding to each panel in the upper part, each of the dots in the bottom panel stands for the point estimate of difference in mean (type 2 - type-1-related), Δ_{mean}. The associated interval is its 95% confidence interval based on bootstrap. In the bottom panel, we draw the horizontal line at zero.

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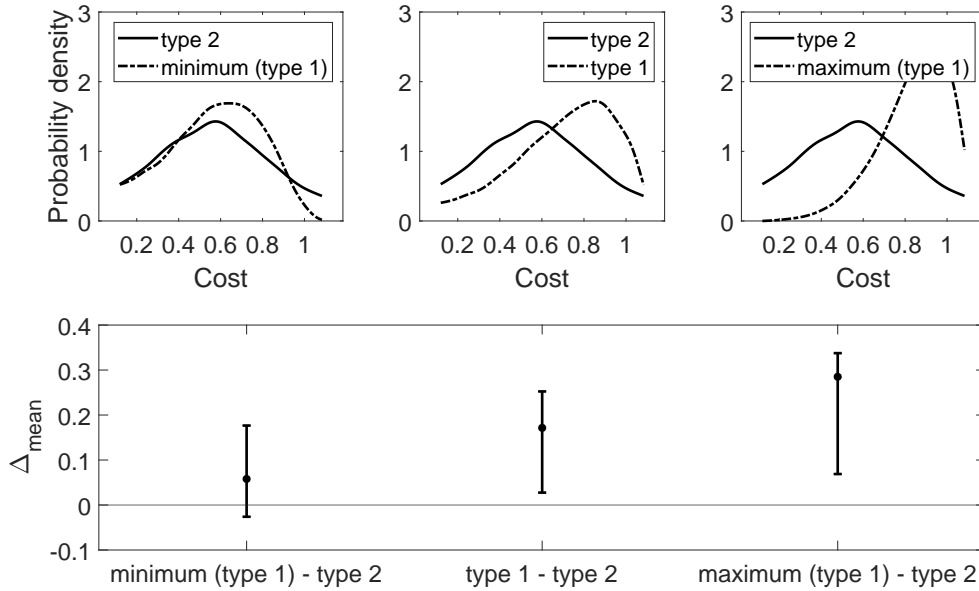
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7 Tables

Table I. Summary of Expenditures by Government Agency

Top 10 Agencies with the Most Auction Expenditures

No.	Agency	Expenditures
1	Housing and Development Board	S$ 4,227,216,158
2	Land Transport Authority	S$ 2,186,562,859
3	Ministry of Health	S$ 1,621,279,711
4	National University of Singapore[32]	S$ 864,642,551
5	National Environment Agency	S$ 751,649,150
6	Ministry of Education	S$ 495,624,541
7	Institute of Technical Education	S$ 383,169,671
8	Ministry of Communications and Information	S$ 351,006,192
9	Public Utilities Board	S$ 291,818,628
10	People's Association	S$ 247,853,304

Top 10 Agencies with Most Auction Expenditures

No.	Agency	Number
1	Ministry of Communications and Information	3,603
2	Ministry of Education Schools	2,731
3	Ministry of Health	892
4	Building and Construction Authority	500
5	Nanyang Technological University	417
6	Housing and Development Board	379
7	Singapore Tourism Board	315
8	People's Association	283
9	Info-communications Media Development Authority	271
10	Institute of Technical Education	267

The table above shows the breakdown of government expenditures on the GeBIZ platform from March 2017 to March 2018, split by government agency. The top panel shows the top 10 agencies with the most expenditure, and the bottom panel shows the top 10 agencies with the most procurement auctions.

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[32]In August 2017, the National University of Singapore left GeBIZ to its own third-party contracting system.

Table II. Summary Statistics

Panel A: Auctions (N = 22,098)

Procurement Item Types	N	Procurement Item Types	N
Administration and Training	6,173	Services	13,611
Services	5,291	Goods	7,887
IT and Telecommunication	2,155	Construction Services	600
Event Organising, Food and Beverages	1,692		
Dental, Medical and Laboratory	1,215		
Facilities Management	1,187		
Transportation	1,183		
Furniture, Office Equipment and AV	974		
Miscellaneous	650		
Workshop Equipment and Services	193		

Variables	Mean	SD
Num. of Bids	6.57	4.59
Num. of Ineffective Bids	0.45	1.22
Winning Bid	708,114	30,760,000
NC^1	0.001	0.025
NC^2	0.087	0.456
NC^3	0.00005	0.007
NC^4	0.005	0.070
Roundedness	0.05	0.13

Panel B: Bids (N = 145,194)

Mean	1,153,705
Standard Deviation	40,533,354
10th Percentile	5,200
25th Percentile	9,240
Median	19,700
75th Percentile	44,950
90th Percentile	101,246

Panel C: Connections (Pair-wise N = 225,295)

Variables	Mean	SD
P(Identical Bids)	0.017	0.114
Num. Joint Participations	2.831	9.605
C_p^1	0.004	0.006
C_p^2	0.076	0.028
C_p^3	0.0004	0.002
C_p^4	0.007	0.008

The table above shows the summary statistics for our main sample. Panel A shows the characteristics and counts of auctions and shows summary statistics for auction-level observations. Panel B shows summary statistics of all the bids. Panel C considers different connections and shows summary statistics for pair-level observations. "N" refers to the number of observations and "SD" stands for standard deviation.

Table III. Number of Ineffective Bids and Ownership Networks

Dependent Variable:	Number of Ineffective Bids								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
NC^1	-0.042				-0.066	0.067	-0.014	-2.109**	
	(0.298)				(0.304)	(0.423)	(0.709)	(0.902)	
NC^2		0.557***			0.557***	0.546***	0.597***	0.724***	0.721***
		(0.038)			(0.038)	(0.042)	(0.051)	(0.041)	(0.175)
NC^3			0.287		0.183				
			(0.305)		(0.258)				
NC^4				0.379***	0.386***	0.307***	0.283***		
				(0.012)	(0.012)	(0.066)	(0.083)		
Sample	Full	Full	Full	Full	Full	$N > 5$	$N > 10$	$N > 20$	$N > 25$
Observations	22,098	22,098	22,098	22,098	22,098	10,491	3,671	337	96
R^2	0.341	0.377	0.341	0.342	0.377	0.401	0.450	0.698	0.869

The table above shows the relation between the number of shareholder connections between participating firms and the number of ineffective bids in an auction. The "Full" sample corresponds to sample of auctions with more than one bidder. NC^k stands for number of connections at k degrees based on current-shareholder connections. Columns (1) through (5) study the impact of connections on the number of ineffective bids, and columns (6) through (9) study the impact of connections based on a sample with more than 5, 10, 20, and 25 bidders, respectively. All regressions include procurer-by-category and number-of-bidder fixed effects, as well as cubic controls of the average roundedness of bids in an auction. The coefficients on the controls are suppressed for space. Standard errors are clustered by procurer and shown in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table IV. Differential Effect of Connections by Asymmetries in Capacity Constraints

Dependent Variable:	Probability of Submitting Identical Bids					
	(1)	(2)	(3)	(4)	(5)	(6)
$X_p =$	Revenue Gap			Asset Gap		
$C_p^2 \times X_p$	-0.028	-0.274***	-0.401***	-0.046**	-0.144***	-0.103
	(0.020)	(0.052)	(0.074)	(0.0205)	(0.045)	(0.065)
C_p^2	0.036	0.310***	0.486***	0.057**	0.210***	0.210***
	(0.025)	(0.048)	(0.074)	(0.026)	(0.045)	(0.059)
X_p	-0.0001	-0.001	0.004	-0.0003	0.001	0.003
	(0.002)	(0.004)	(0.007)	(0.002)	(0.004)	(0.007)
Observations	6,592	860	374	6,592	860	374
Num. Joint Participations:	$N \geq 1$	$N \geq 5$	$N \geq 10$	$N \geq 1$	$N \geq 5$	$N \geq 10$
R^2	0.000	0.047	0.107	0.001	0.028	0.044

The table above shows the differential effects of connections on the probability of submitting identical bids. In the first and fourth columns, we use all the pairs. In the second and fifth columns, we focus on pairs that jointly bid at least five times. In the third and sixth columns, we focus on pairs that jointly bid at least 10 times. X_p stands for asymmetries in capacity constraints. Revenue gap is the absolute difference in the two firms' revenues normalized by the average revenue of the two firms. Asset gap is the absolute difference in the two firms' asset sizes normalized by the average asset size of the two firms. C_p^k stands for connections of k degrees based on current-shareholder connections. Standard errors are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table V. Winning Bids and Number of Ineffective Bids

Dependent Variable:	Winning Bid Normalized by Mean Bid					
Category	All		Goods		Services	
Type	Uninstrumented	IV	Uninstrumented	IV	Uninstrumented	IV
	(1)	(2)	(3)	(4)	(5)	(6)
Num. Ineffective Bids	0.045***	0.045*	0.041***	0.176**	0.049***	-0.006
	(0.006)	(0.025)	(0.007)	(0.078)	(0.003)	(0.025)
Num. Bidders	-0.062***	-0.062***	-0.057***	-0.062***	-0.059***	-0.065***
	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.010)
Num. Bidders^2	0.002***	0.002***	0.001***	0.001***	0.002***	0.002***
	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0003)
First Stage F-Statistic		26.335		31.924		8.995
Observations	11,617	11,617	4,555	4,555	7,062	7,062
R^2	0.461	0.461	0.481	0.392	0.440	0.471

The table above shows the relation between the normalized winning bid and the number of ineffective bids using an auction-level panel restricted to auctions in which empirically, the winning bid was the lowest bid in the auction. Columns (1) and (2) present the results for all eligible auctions, columns (3) and (4) only use auctions for goods, and columns (5) and (6) only use auctions for services. Columns (1), (3), and (5) present results for the uninstrumented number of ineffective bids, whereas columns (2), (4), and (6) present results from the instrumental variable (IV) method whereby the number of ineffective bids is instrumented by second- and fourth-degree connections. All regressions include procurer-by-category fixed effects, quadratic controls for the number-of-bidders, as well as cubic controls of the average roundedness of bids in an auction, whose coefficients are suppressed for space. Standard errors are clustered by procurer and shown in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table VI. Collusive Versus Competitive Bids

Dependent Variable:	The Value of a Bid	
	(1)	(2)
Identical	-33,480.0***	-27,028.2**
	(9,758.4)	(13,470.1)
Observations	135,001	44,719
R^2	0.509	0.976

The table above shows the effect of being an identical bid on the value of a bid. In the first column, we use all the effective bids in our sample auctions. In the second column, we focus on the effective bids of auctions in which lowest bidders win. All regressions include auction fixed effects. Standard errors are in parentheses and clustered at auction level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
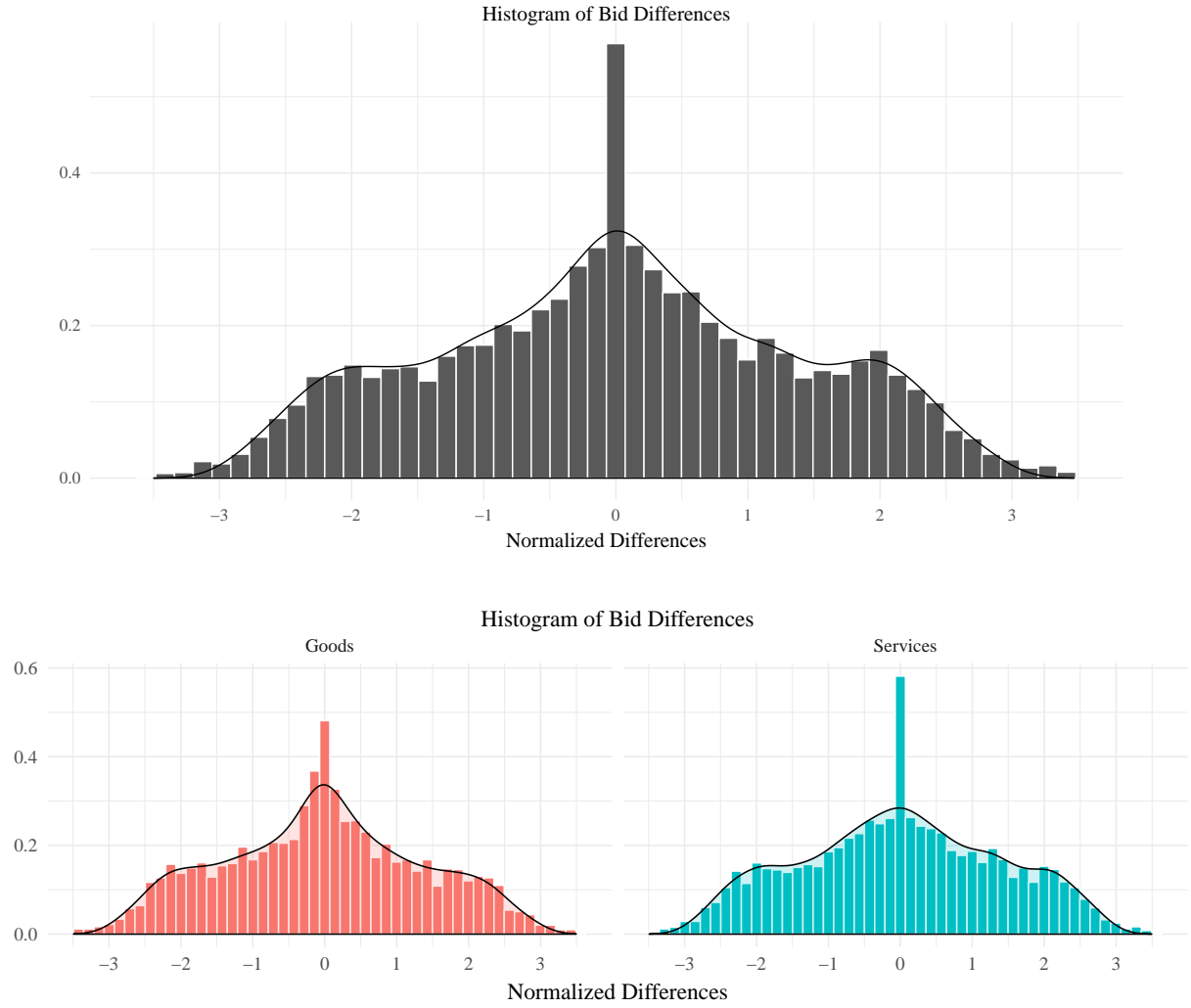
8 Figures

Figure I. Connections
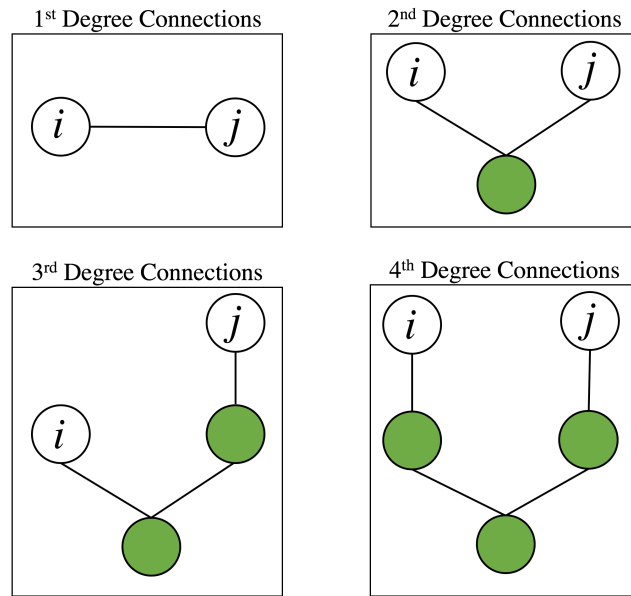
We show the sample connections below for some arbitrary firms i and j. For second-degree connections, the shaded node may be an individual or company. For third-degree connections, the bottom shaded node may be a firm or company, and the top shaded node must be a company. For fourth-degree connections, the bottom node may be an individual or company and the top two shaded nodes must be companies.

Figure II. Distribution of Bid Differences

The figure above shows the distribution of differences from a randomly selected pair of bids from auctions with at least three bids, where bids are normalized by the mean of all bids. The top panel shows the bid differences across all valid auctions, whereas the second panel splits it into mutually exclusive categories of Goods versus Services. In all the histograms, a mass appears to exist at the bid difference equal to zero.

Figure III. The Effect of Excluding a Second-Degree Connection on Auction Outcomes
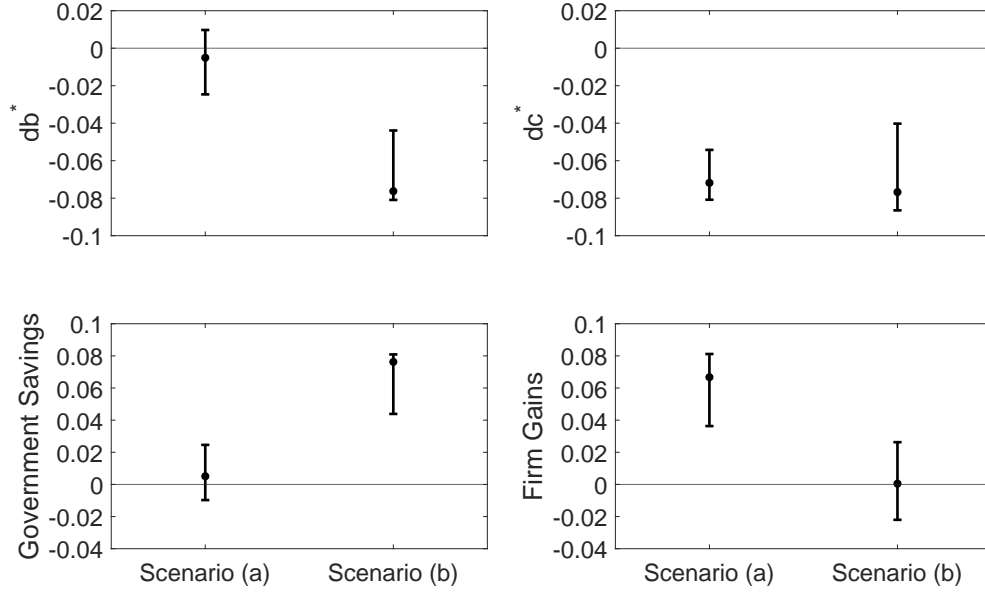
In this figure, we use the reduced-form evidence in Table III column (2). In every panel, black dots stand for point estimates and their associated intervals represent corresponding 95% confidence intervals based on bootstrap. In every panel, we draw the horizontal line at zero. We present the estimated effect of excluding a second-degree current-shareholder connection on the winning bid for each scenario in the top left panel, the effect on the winner's cost in the top right panel, the corresponding government savings in the bottom left panel, and the corresponding firm gains in the bottom right panel.

Figure IV. The Effect of Excluding a Fourth-Degree Connection on Auction Outcomes
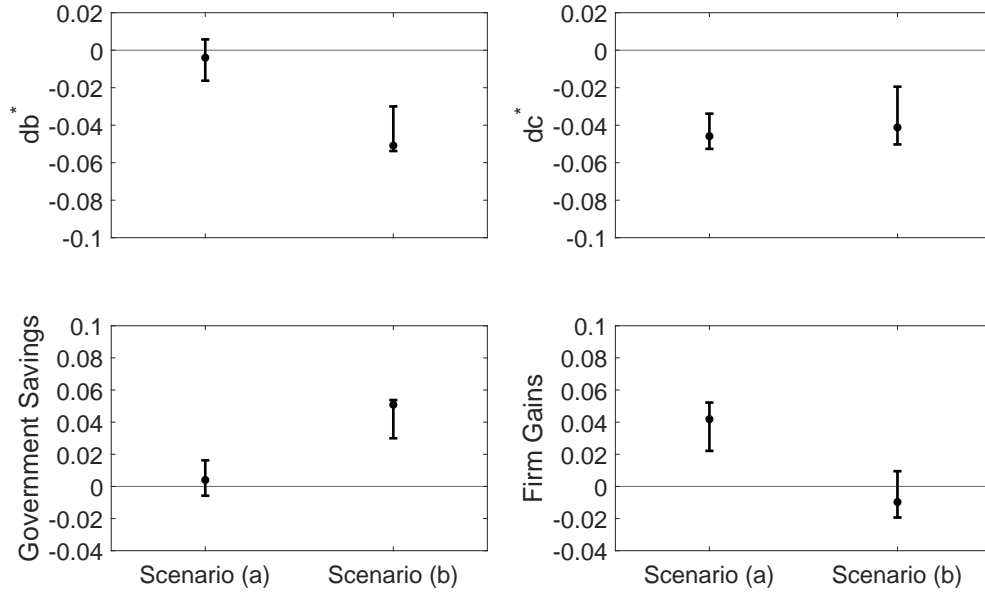
In this figure, we use the reduced-form evidence in Table III column (4). In every panel, black dots stand for point estimates and their associated intervals represent corresponding 95% confidence intervals based on bootstrap. In every panel, we draw the horizontal line at zero. We present the estimated effect of excluding a fourth-degree current-shareholder connection on the winning bid for each scenario in the top left panel, the effect on the winner's cost in the top right panel, the corresponding government savings in the bottom left panel, and the corresponding firm gains in the bottom right panel.